Influence of Electrical Noise on Temporal Optical Phase Reconstruction Using Transport-of-Intensity Equation

Masayuki Matsumoto

Faculty of Systems Engineering, Wakayama University

Wakayama, Japan

WB2.1

2020 IEEE Photonics Society Summer Topicals Meeting Series

July 15, 2020

Outline

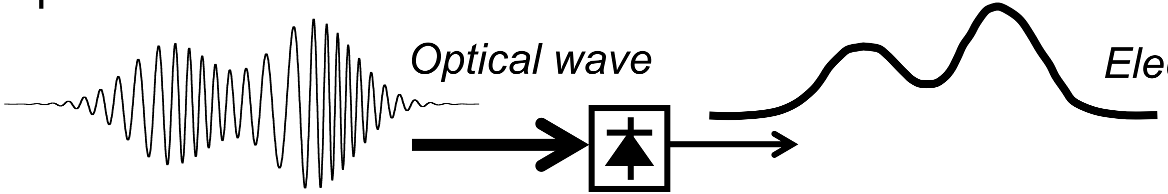
1. Background
2. Optical phase reconstruction using transport-of-intensity equation
   - Principle and features
   - Examples of phase retrieval using temporal TIE
3. Numerical simulation of QPSK and 16-QAM signal reconstruction
   - Required carrier-to-signal power ratio
   - Reasons of failure in signal reconstruction
   - Effects of location of carrier in signal spectrum
4. Influence of electrical noise on signal reconstruction
   - Different difference schemes
5. Conclusion

M. Matsumoto, “Optical signal phase reconstruction based on temporal transport-of-intensity equation”, J. Lightwave Technol., Early Access, DOI: 10.1109/JLT.2020.2995640 (2020).

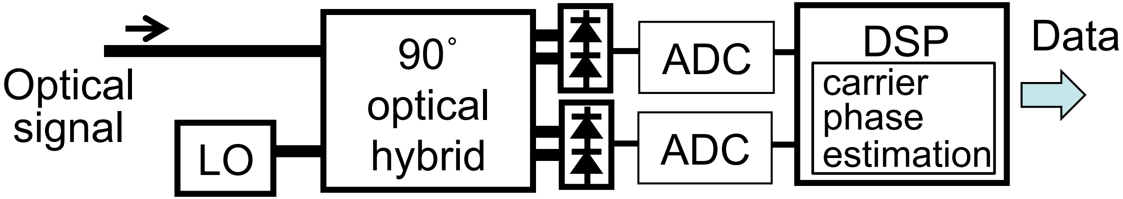
Background

In optical data transmission, information is encoded on amplitude and phase of optical waves. Utilization of *both amplitude and phase* is required for highly spectral and power efficient signal transmission.

Amplitude or power of optical waves can be detected by photo-detectors. But phase is lost at the photo-detection.



Coherent receivers can detect optical phase but with complicated receiver structures.



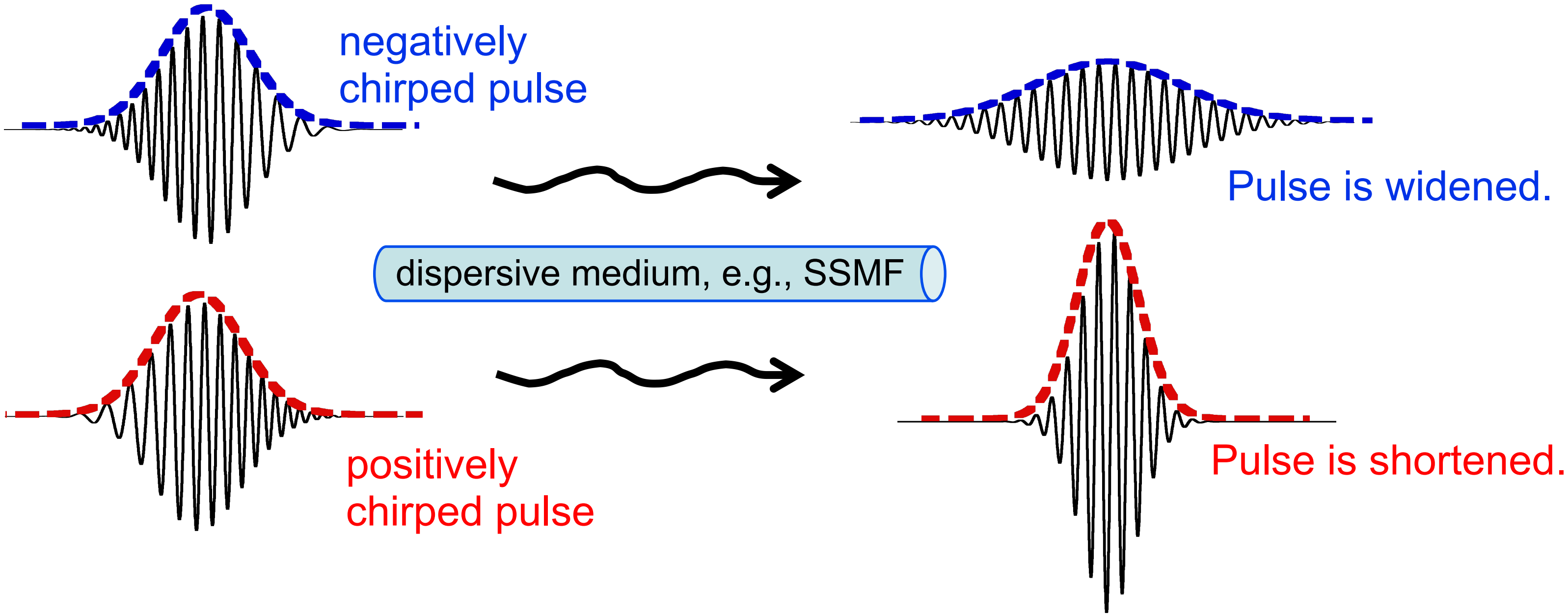
Background

Phase retrieval using direct detection is desired and a number of schemes have been studied:

- ❖ Self-coherent receivers with iterative SSBI mitigation
- ❖ Stokes-vector receivers
- ❖ Kramers-Kronig receiver
- ❖ Phase reconstruction using linear optical filters
- ❖ Iterative phase retrieval by solving generalized optimization problems

- ❖ Phase reconstruction utilizing dispersive signal transmission:
  1. Iterative phase retrieval based on Gerchberg-Saxton and related algorithms
  2. Non-iterative direct phase retrieval based on TIE

Principle of phase retrieval based on TIE



- ✧ Dispersive signal distortion depends on the phase before dispersion.
- ✧ Phase of optical signals can be calculated when amplitude or power waveforms both before and after dispersion are known. In the method based on TIE, the derivative of the intensity waveform with respect to distance is used for the phase calculation.

Principle of phase retrieval based on TIE

Transport-of-Intensity Equation (TIE)

Phase retrieval based on TIE has been used in measuring two-dimensional phase distribution, or wave fronts, in electron microscopy, chrystallography, and X-ray and optical imaging.

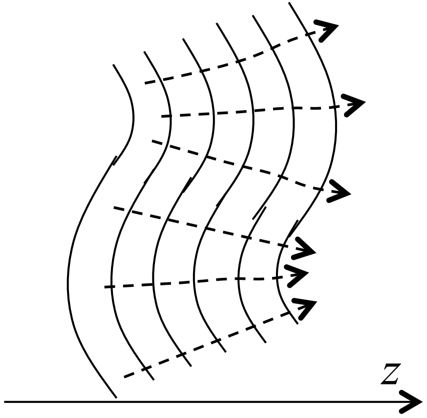
$$\nabla_{\perp}^{2} u + 2ik\frac{\partial u}{\partial z} = 0$$

Helmholtz equation in paraxial approximation for spatially-propagating stationary waves

$$u(x,y,z) = \sqrt{I(x,y,z)}\exp(i\phi(x,y,z))$$

$$\boxed{\nabla_{\perp}\cdot(I\nabla_{\perp}\phi) = -k\frac{\partial I}{\partial z}}$$

TIE



positive (negative) curvature of wave fronts corresponds to decrease (increase) in intensity in z direction.

M. R. Teague, JOSA, vol.73, no.11, 1434(1983).

Principle of phase retrieval based on TIE

The TIE has also been applied to phase reconstruction of temporal optical signals.

Complex amplitude of optical signals in a dispersive medium

$$\frac{\partial f}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 f}{\partial t^2} = 0$$

β_2 : group-velocity dispersion

$$f(t,z) = \sqrt{P(t,z)}\exp(i\phi(t,z))$$

$$\boxed{\frac{\partial}{\partial t}\left(P\frac{\partial \phi}{\partial t}\right) = \frac{1}{\beta_2}\frac{\partial P}{\partial z}}$$

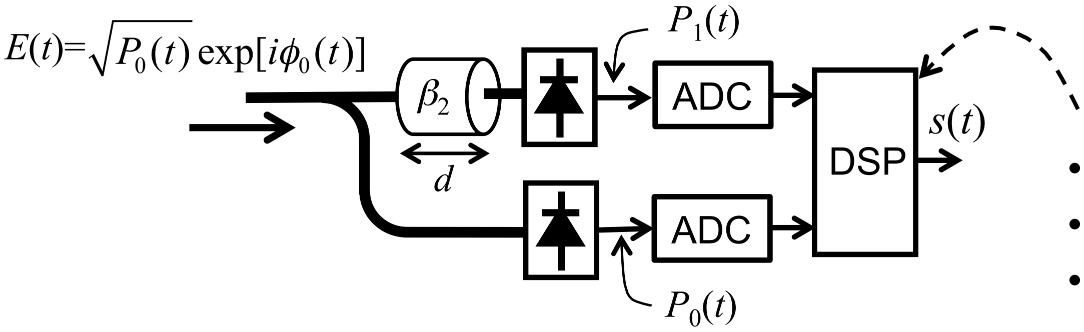
One-dimensional temporal TIE

$\phi(t,z)$ is obtained by solving this equation when $P(t,z)$ and $\partial P/\partial z(t,z)$ are known.

Previously, the one-dimensional temporal TIE has been used for characterizing short optical pulses in which phase across the pulse is calculated from its intensity waveforms.

C. Dorrer, Opt. Lett., vol. 30, no. 23, pp. 3237-3239 (2005).
C. Cuadrado-Laborde et al., Opt. Lett., vol. 39, 8, 598 (2014).

Receiver structure and solution algorithm



- Solution of TIE $\frac{\partial}{\partial t}\left(P\frac{\partial \phi}{\partial t}\right)=\frac{1}{\beta_2}\frac{\partial P}{\partial z}$
- Removal of carrier
- Equalization

- Right-hand side of the TIE is approximated by $\left[P_1(t)-P_0(t)\right]/(\beta_2 d)$.
- $P_0(t)$ is used as P in the left-hand side of the TIE.
- FFT and IFFT are used to solve the TIE.
- $P_0(t), P_1(t)$, and $\phi_0(t)$ are sampled at $t=0, \Delta t, 2\Delta t, \ldots, (M-1)\Delta t$, where $T=M\Delta t$ is the time duration of a signal block.

$$\phi_0 = \boldsymbol{F}^{-1}\boldsymbol{D'}\boldsymbol{F}\boldsymbol{P'}\boldsymbol{F}^{-1}\boldsymbol{D'}\boldsymbol{F}\boldsymbol{g} - (a_2/a_3)\boldsymbol{F}^{-1}\boldsymbol{D'}\boldsymbol{F}\boldsymbol{P'}\boldsymbol{1} + c_2\boldsymbol{1}$$

$\boldsymbol{F}(\boldsymbol{F}^{-1})$: matrices representing FFT(IFFT).

$\boldsymbol{P'}$: diagonal matrix composed of $P_0(t_i)$

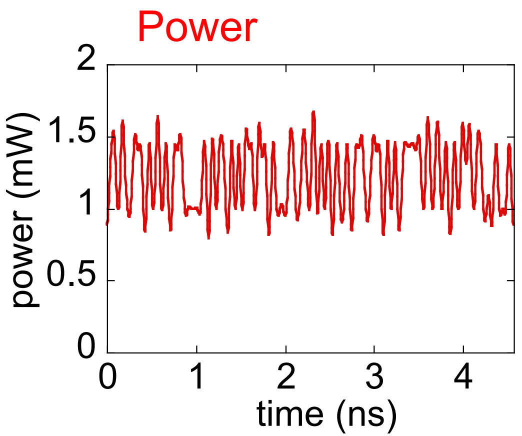
$\boldsymbol{g}$: vector representing RHS of TIE.

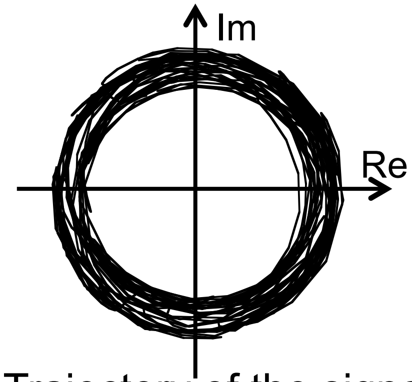
$\boldsymbol{D'}$: diagonal matrix representing integration

Phase discontinuity by an integer multiple of 2π can be incorporated.

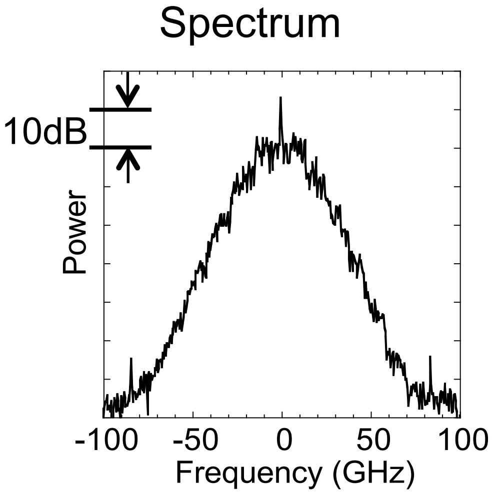
Examples of phase reconstruction 1

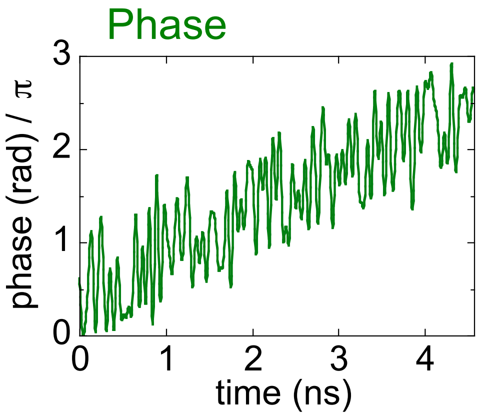
An arbitrary signal having power well above zero





Trajectory of the signal
in complex plane





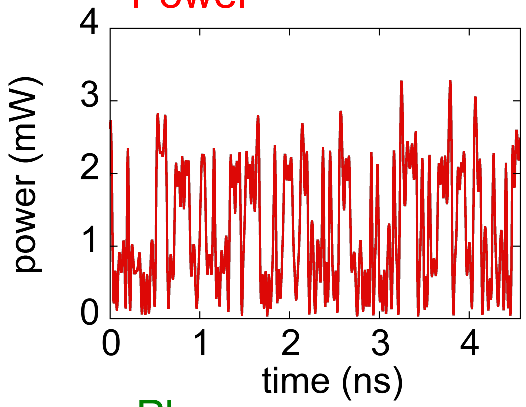
Accurate reconstruction is achieved. (Dispersion Dd=10ps/nm)

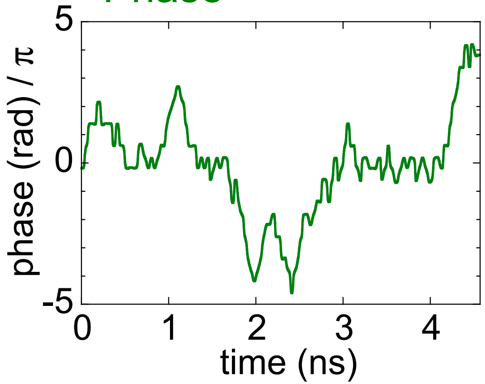
Reconstruction error $\int |\hat{E}(t) - E(t)|^2 \, dt \Big/ \int |E(t)|^2 \, dt$ = -29.5dB

RMS of phase error = 0.033rad

Examples of phase reconstruction 2

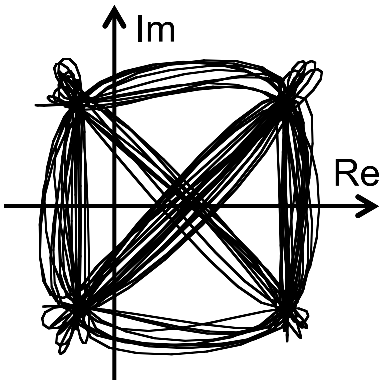
Slightly-biased Nyquist (α=0.6) QPSK signal (back-to-back)



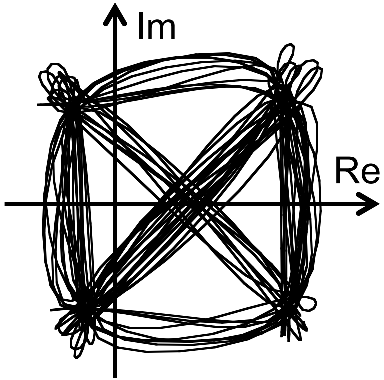


28GBaud, 128 symbols Dispersion Dd=10ps/nm

CSPR=-6dB, OSNR$_e$=30dB



original signal



reconstructed signal

Reconstruction error $\int |\hat{E}(t) - E(t)|^2 \, dt \Big/ \int |E(t)|^2 \, dt$ = -25.8dB

RMS of phase error = 0.054rad

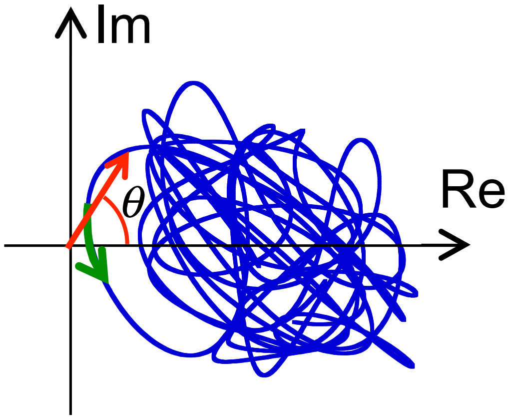
✓ The TIE-based method can reconstruct signals that do not satisfy the minimum phase condition.

QPSK and 16QAM signal reconstruction

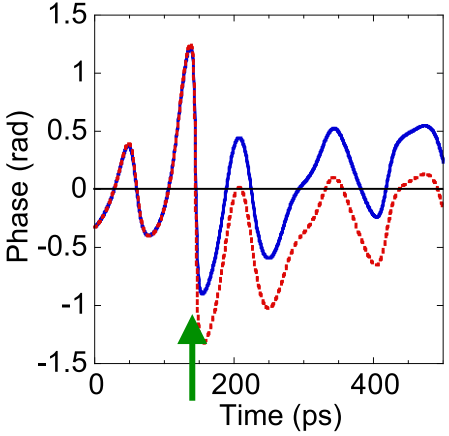
Biased Nyquist QPSK and 16QAM signals

$$E(t) = A_0 + s(t)\, e^{-i2\pi Bt}$$ (B: frequency difference between carrier and signal)

The addition of the carrier to the signal is needed so that $P(t)=|E(t)|^2$ stay positive well above zero.



Complex amplitude passes close to the origin at t=140ps.



Trajectory of 28GBaud Nyquist 16QAM signal

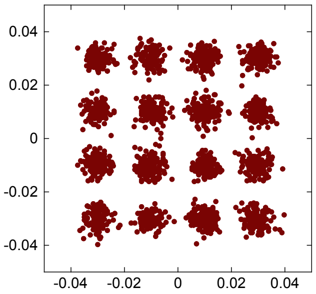
CSPR (Carrier to Signal Power Ratio)=5.5dB

Phase variation in time

Blue curve: phase of 16QAM signal

Red curve: phase reconstructed by TIE

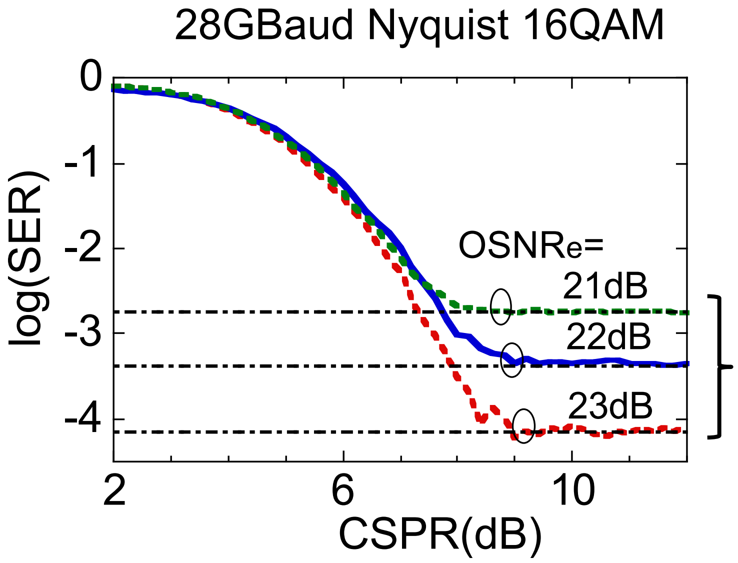
QPSK and 16QAM signal reconstruction



Constellation after carrier is subtracted

Distance L=0km

Dd=10ps/nm



$$OSNR_e = P_s/(NB_{\mathrm{ref}})$$

(Signal power does not include carrier power.)

Theoretical SER

$$SER_{16\mathrm{QAM}} = 3r - \frac{9}{4}r^2$$

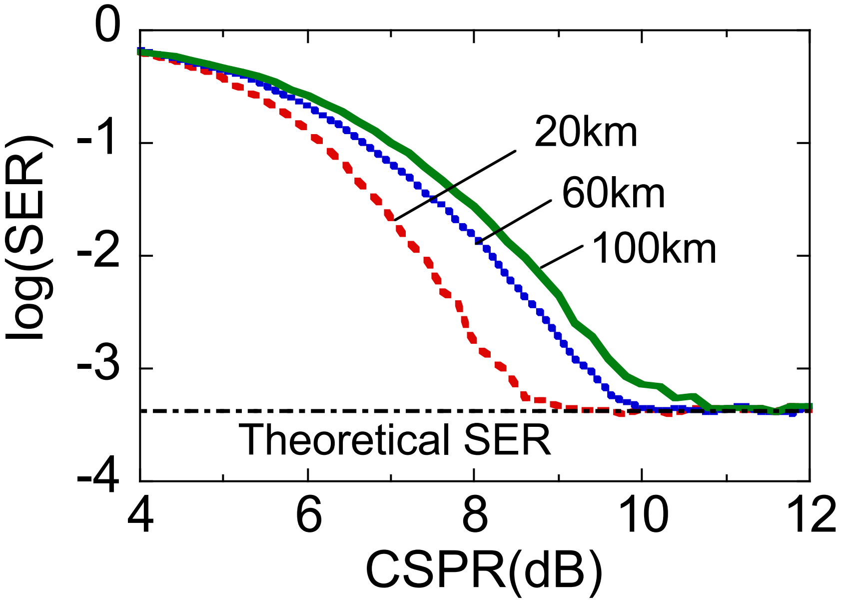
$$r = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{OSNR_{\mathrm{e}}B_{\mathrm{ref}}}{10B_{\mathrm{ASE}}}}\right)$$

Signal reconstruction error grows as the CSPR is decreased to smaller than 8~9dB.

The value of 8~9dB CSPR is related to the peak-to-average power ratio (PAPR) of the signal before the carrier is added.

Influence of transmission distance

28GBaud Nyquist 16QAM signals transmitted over 20-100km SSMF.



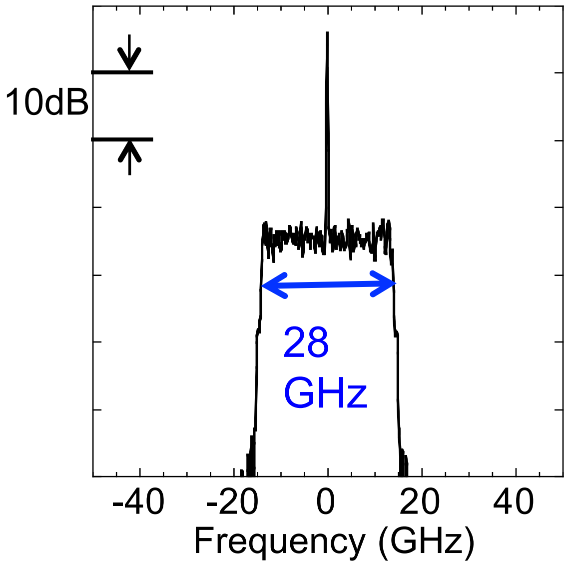
Dd=10ps/nm

$OSNR_e$=22dB

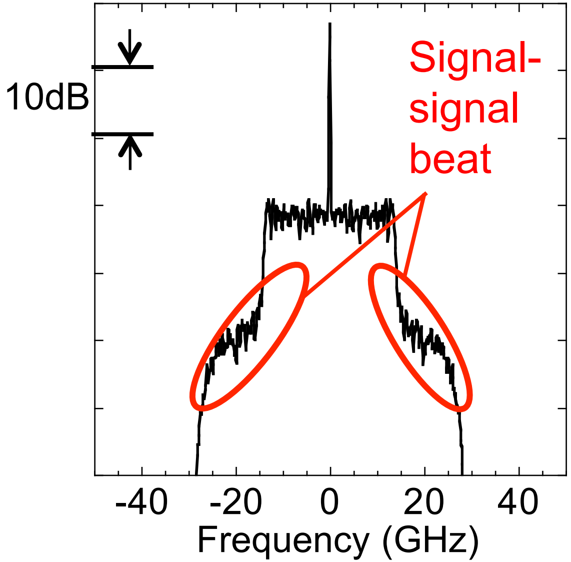
Larger CSPR is required for longer transmission distance. This is because PAPR grows as the distance is increased.

Sampling frequency

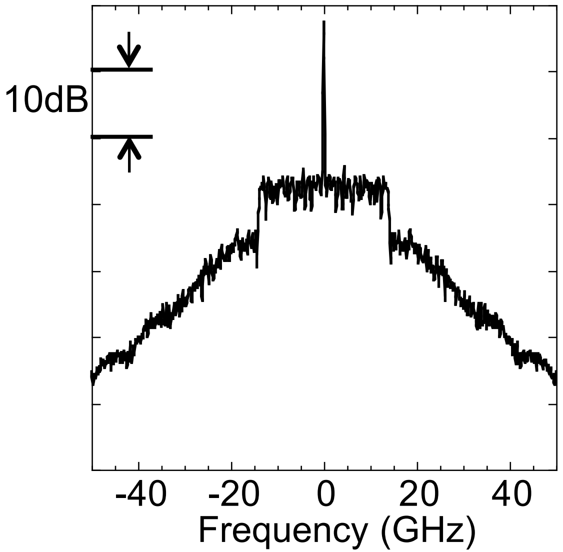
28GBaud Nyquist QPSK signal



Optical spectrum



Spectrum of detected current $P(t)$



Spectrum of $1/P(t)$

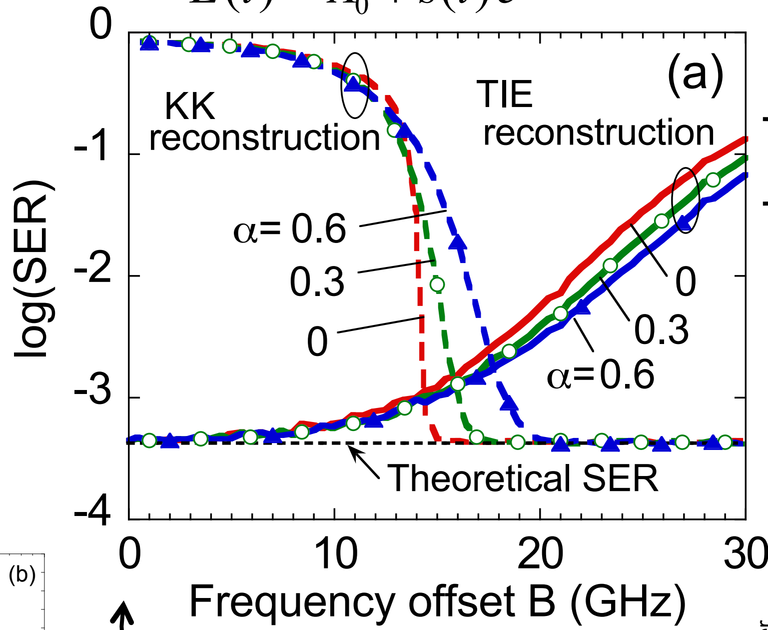
Electrical bandwidth is extended by the operation of $P(t)^{-1}$

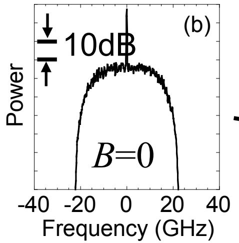
Sampling frequency of 4 samples/symbol is needed in the electrical signal processing.

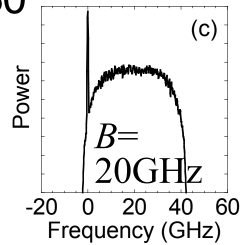
Frequency offset between carrier and signal

28GBaud Nyquist 16QAM with different roll-off factors

$$E(t) = A_0 + s(t)e^{-i2\pi Bt}$$







$OSNR_e$=22dB, $CSPR$=11dB

Transmission distance L=100km

Smallest SER is obtained when the carrier is located at the center of the signal spectrum for TIE-based reconstruction.

Reconstruction fails when the carrier is located inside the signal spectrum for Kramers-Kronig algorithm.

Influence of electrical noise

The forward finite difference is written in the absence of noise as

$$\frac{P(z_0+\Delta z)-P(z_0)}{\Delta z}=\left.\frac{\partial P}{\partial z}\right|_{z0}+O(\Delta z)$$

Smaller approximation error is obtained as $\Delta z \rightarrow 0$.

When noise is considered, uncorrelated noise is added to $P(z_0)$ and $P(z_0+\Delta z)$.

$$\frac{P(z_0+\Delta z)+n_1-P(z_0)-n_0}{\Delta z}=\left.\frac{\partial P}{\partial z}\right|_{z0}+\frac{n_1-n_0}{\Delta z}+O(\Delta z)$$

This means that the influence of noise is enhanced for smaller Δz.

The noise issue in the TIE-based phase reconstruction has been recognized and well studied in the field of optical imaging and electron microscopy*.

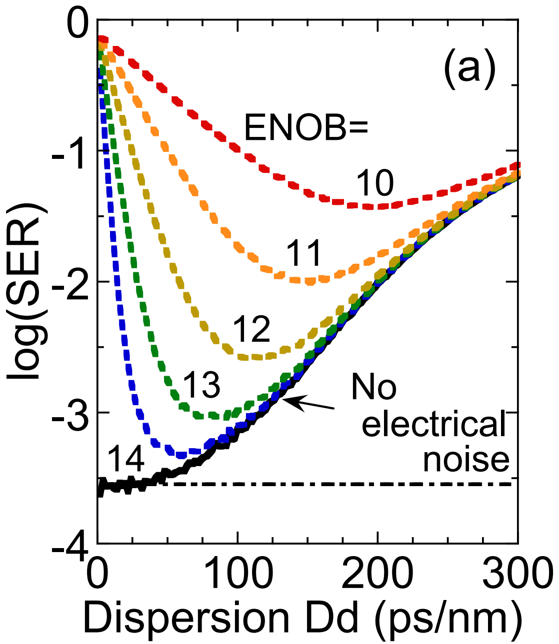
* See for example, L. Waller et al., "Transport of intensity phase-amplitude imaging with higher order intensity derivatives", Opt. Exp. 18, 12, 12552 (2010).

Influence of electrical noise

Uncorrelated Gaussian noise $n_0(t)$ and $n_1(t)$ are added to $P_0(t)$ and $P_1(t)$, respectively.

Magnitude of the noise is quantified by $SNR = \left\langle [P_i(t) - \langle P_i(t) \rangle]^2 \right\rangle / \left\langle n_i^2(t) \right\rangle$, i=0 or 1.

$ENOB$ is defined as $ENOB = (SNR[\mathrm{dB}]\text{-}1.76)\ /\ 6.02$



28GBaud Nyquist QPSK

Transmission distance = 100km

$OSNR_e$=15dB, $CSPR$=11dB

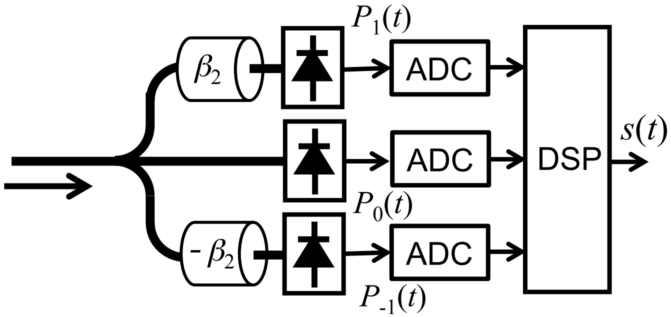
For SER < 10^{-3},
$ENOB$=13 or SNR=80dB is needed.

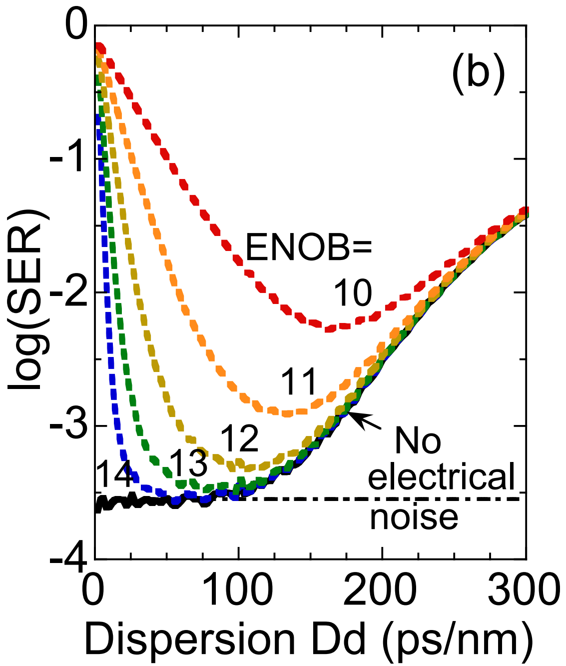
Temporal TIE signal reconstruction is sensitive to electrical noise.

Different difference schemes

Forward difference $[P(z_0+\Delta z)-P(z_0)]/\Delta z = \partial P/\partial z\big|_{z0} + O(\Delta z)$

Central difference $[P(z_0+\Delta z)-P(z_0-\Delta z)]/(2\Delta z) = \partial P/\partial z\big|_{z0} + O((\Delta z)^2)$



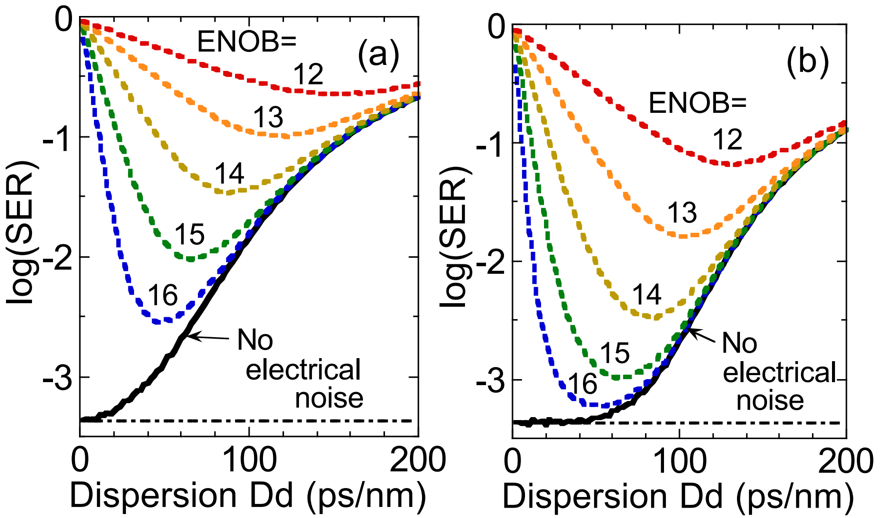


Required ENOB for SER<10^{-3} is reduced to 11.5 by the use of central difference scheme.

Influence of electrical noise to 16QAM reconstruction

28GBaud Nyquist 16QAM, $OSNR_e$=22dB, $CSPR$=11dB

Transmission distance = 100km



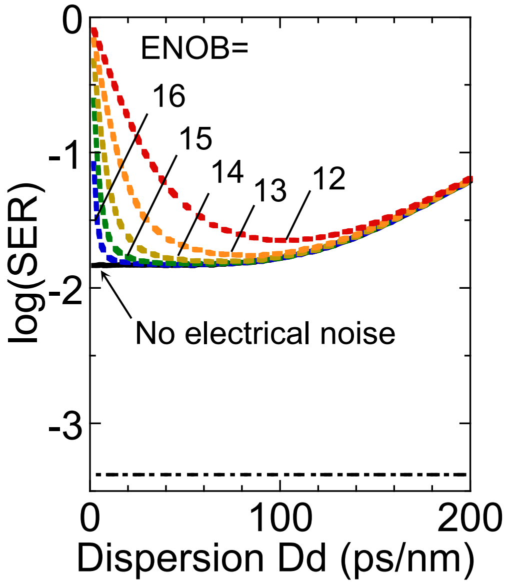
Forward difference scheme

Central difference scheme

Minimum SER @ENOB=12

← 0.066

0.023 →



Lowest frequency components in the Fourier expansion are disregarded in the solution of TIE.

Other various methods for noise reduction that have been developed in non-iterative TIE-based phase imaging are expected to be useful also in temporal phase reconstruction

Conclusion

Phase reconstruction of high-speed optical signals based on temporal transport-of-intensity equation (TIE) is described.

- Signals are retrieved by non-iterative direct calculation.
- Signals are not restricted to SSB ⟶ high flexibility in signal format to be used
- Sampling speed for signal processing is ~ 4 samples/symbol.
- Required carrier power is a few dB larger than Kramers-Kronig reconstruction.
- Sensitive to electrical noise at signal detection.

Future studies:

- Noise tolerant detection using multiple detectors
- Schemes for reducing carrier power