On Automatic Generation of Escher-like Metamorphosis

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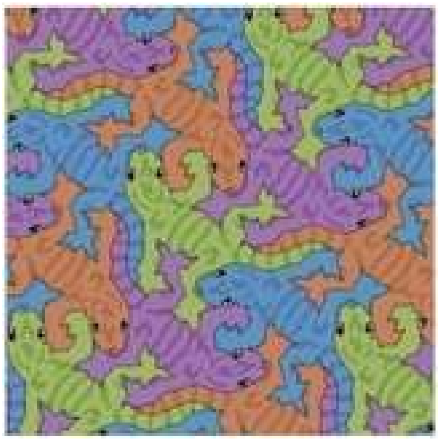
Background

- **Tiling** (or **tessellation**) is the operation of covering a plane without gaps or overlaps using a finite set of flat shapes (tiles).
- Dutch artist **M. C. Escher** is known as an artist who pioneered **tiling art**.


tiling art by Escher

Background

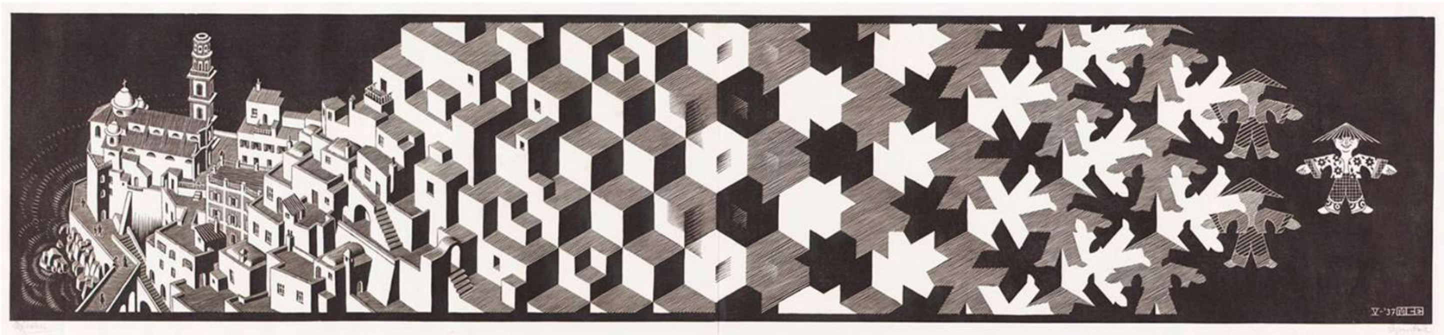
- Recent **AI** generates paintings that are often difficult to distinguish from products by human artists.
- There are algorithms for generating tiling art.



Tiling art automatically generated by computer

Motivation

- **Metamorphosis** by Escher is the work in which a figure in a tile continuously transforms to a different figure.
- The state-of-the-art **generative AI** does not produce Escher-style Metamorphosis using a simple prompt.



Escher's Metamorphosis

The output of Stable Diffusion using the prompt: "Escher-style metamorphosis".

Purpose

- We develop an algorithm that **automatically generates Escher-like Metamorphosis**.
- We reproduce morphing images in Metamorphosis by Escher, and also generate **original Escher-like Metamorphosis** using color images.
- The system could be used for **assisting artistic works and product designs**.

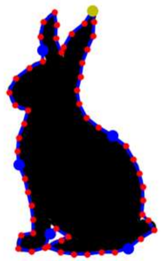
Escherization Problem

Given an input figure **S**, output a new figure **T** satisfying the two conditions:

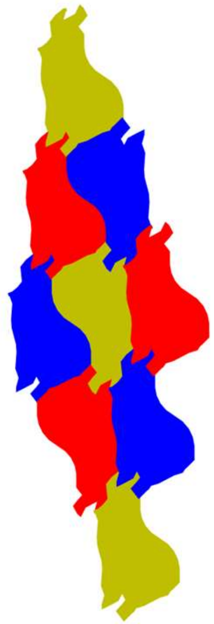
① **T is as close as possible to S**;

② **Copies of T form a tiling of a plane**.

input **S**



output **T**



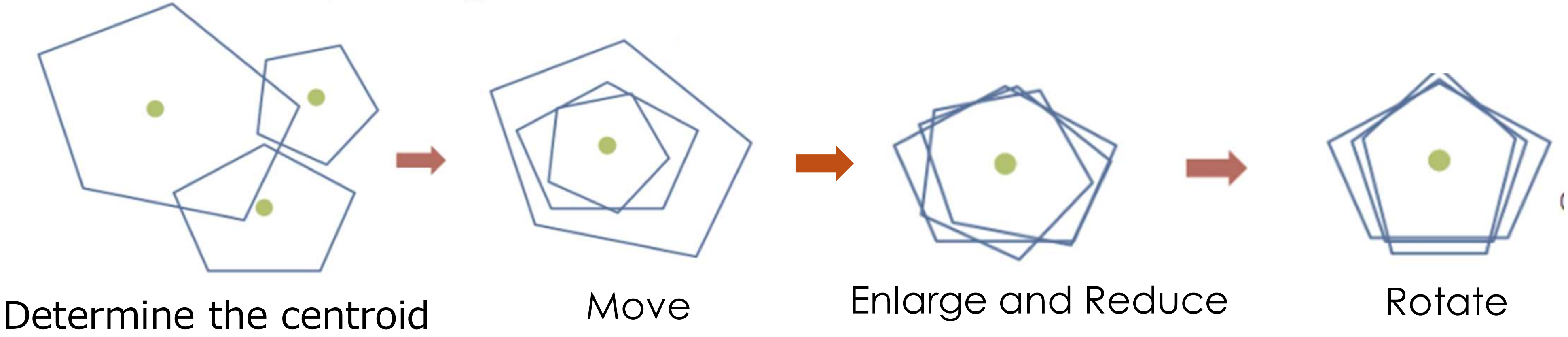
C. S. Kaplan and D. H. Salesin. ``Escherization”.

In: Proc. 27th Annual Conf. Computer Graphics and Interactive Techniques (2000)

Escherization Problem

① A new figure T is as close as possible to an input figure S

➡ Find a shape with the smallest **Procrustes distance** from the input figure.

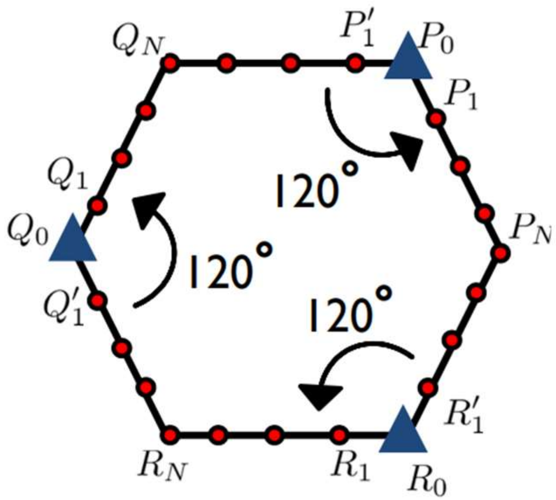


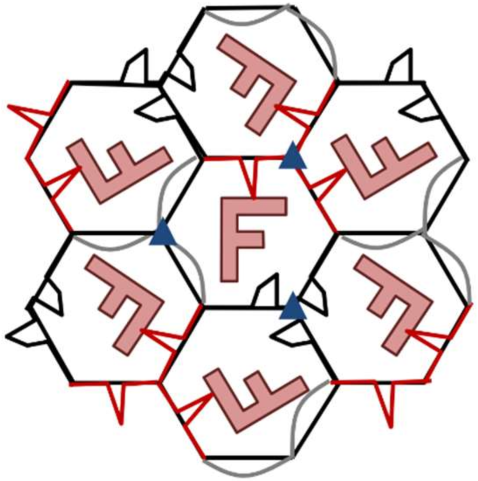
H. Koizumi and K. Sugihara. ``Maximum eigenvalue problem for Escherization”. Graphs and Combinatorics 27 (2011)

Escherization Problem

② **Copies of a new figure T form a tiling of a plane**

➡ Find a figure (hexagon) satisfying the constraints of **isohedral tiling (IH7)**.





$$\begin{cases} S(P'_k - P_0) = P_k - P_0 \, (k = 1, \dots, N-1) \\ S(Q'_k - Q_0) = Q_k - Q_0 \, (k = 1, \dots, N-1) \\ S(R'_k - R_0) = R_k - R_0 \, (k = 1, \dots, N-1) \end{cases}$$

S is the 120° rotation matrix

H. Koizumi and K. Sugihara. ``Maximum eigenvalue problem for Escherization”. Graphs and Combinatorics 27 (2011)

Escherization Problem

① A new figure **T** is as close as possible to an input figure **S**;

➡ **Procrustes distance**

② Copies of a new figure **T** form a tiling of a plane.

➡ **Isohedral tiling (IH7)**

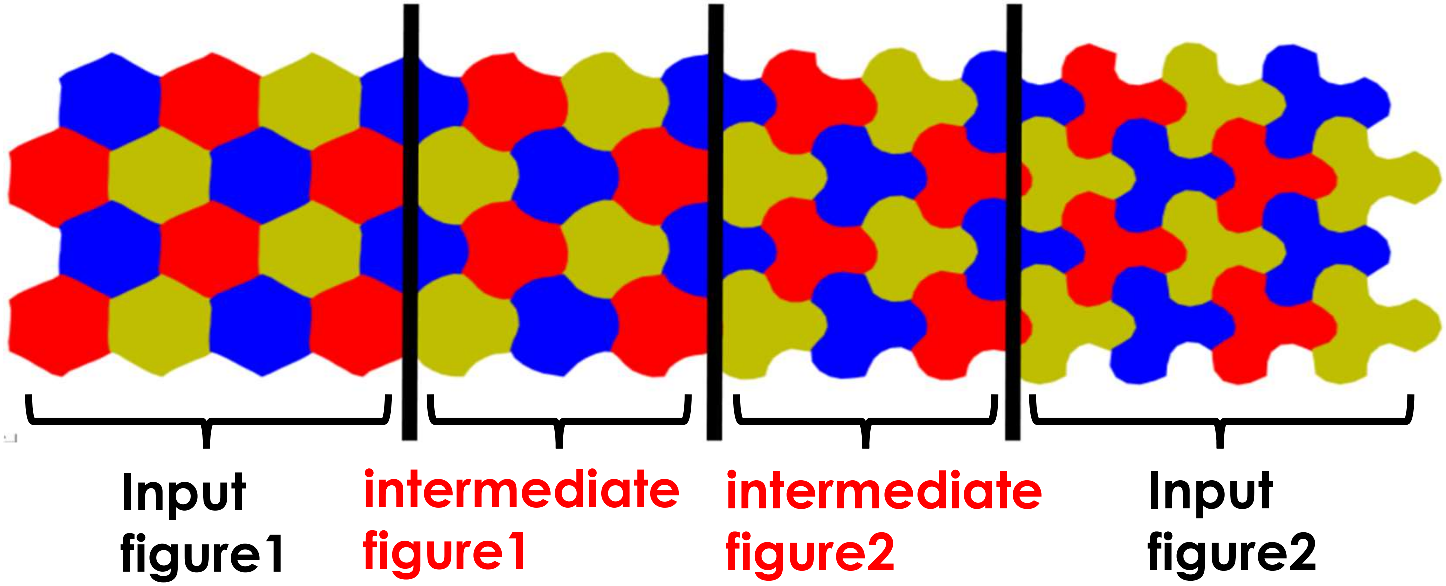
⬇

The tile figure **U** is represented as $u = \mathrm{BB}^T w$
using the principal eigenvector $\mathrm{B}^T w$.

H. Koizumi and K. Sugihara. ``Maximum eigenvalue problem for Escherization''. Graphs and Combinatorics 27 (2011)

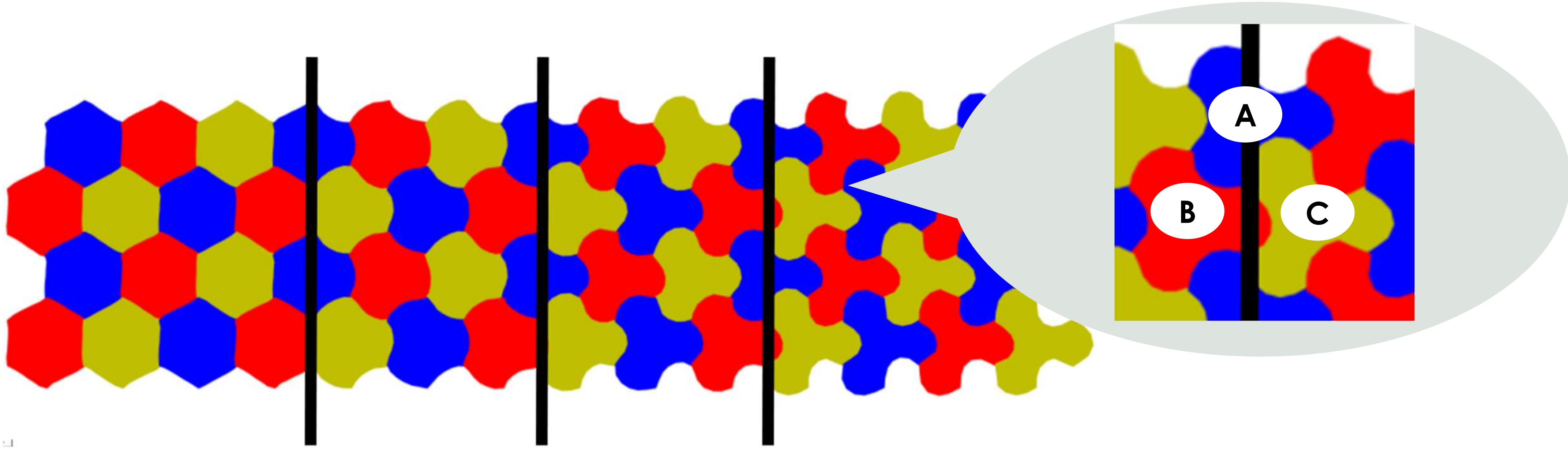
Proposed Method: Tiling for Metamorphosis

1. Divide the plane into blocks and perform the same tiling based on IH7 within each block.
2. Introduce **intermediate figures** that represent the form between the left and right figures.



Proposed Method: Boundary Figure

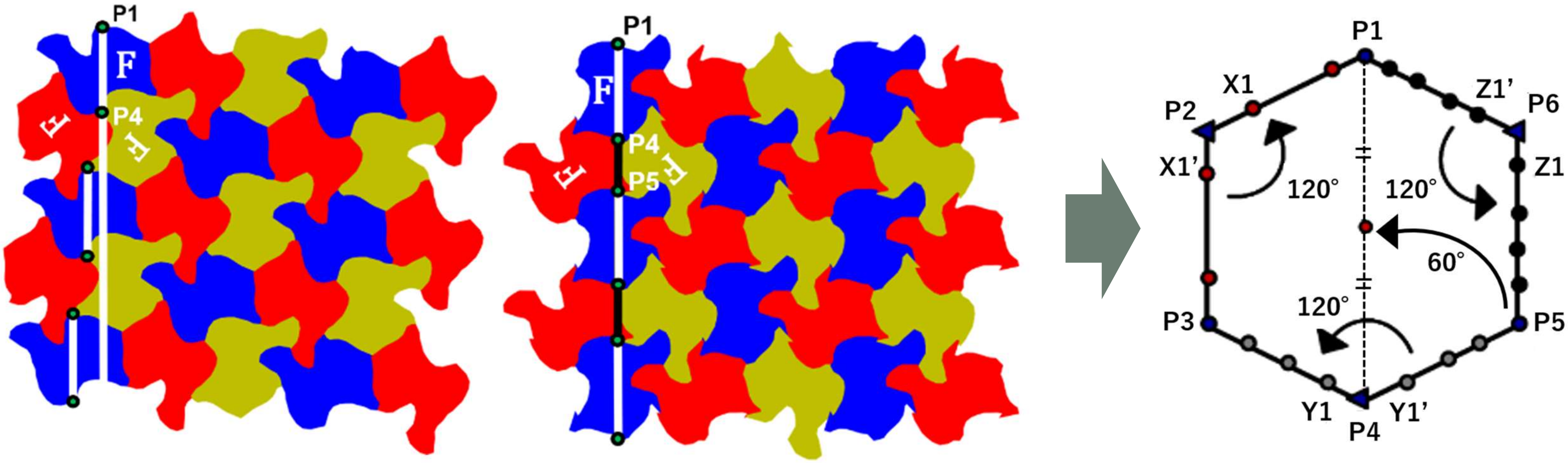
Introduce **boundary figures** to be placed on the boundary line and add **boundary constraints** to effectively connect the boundaries.



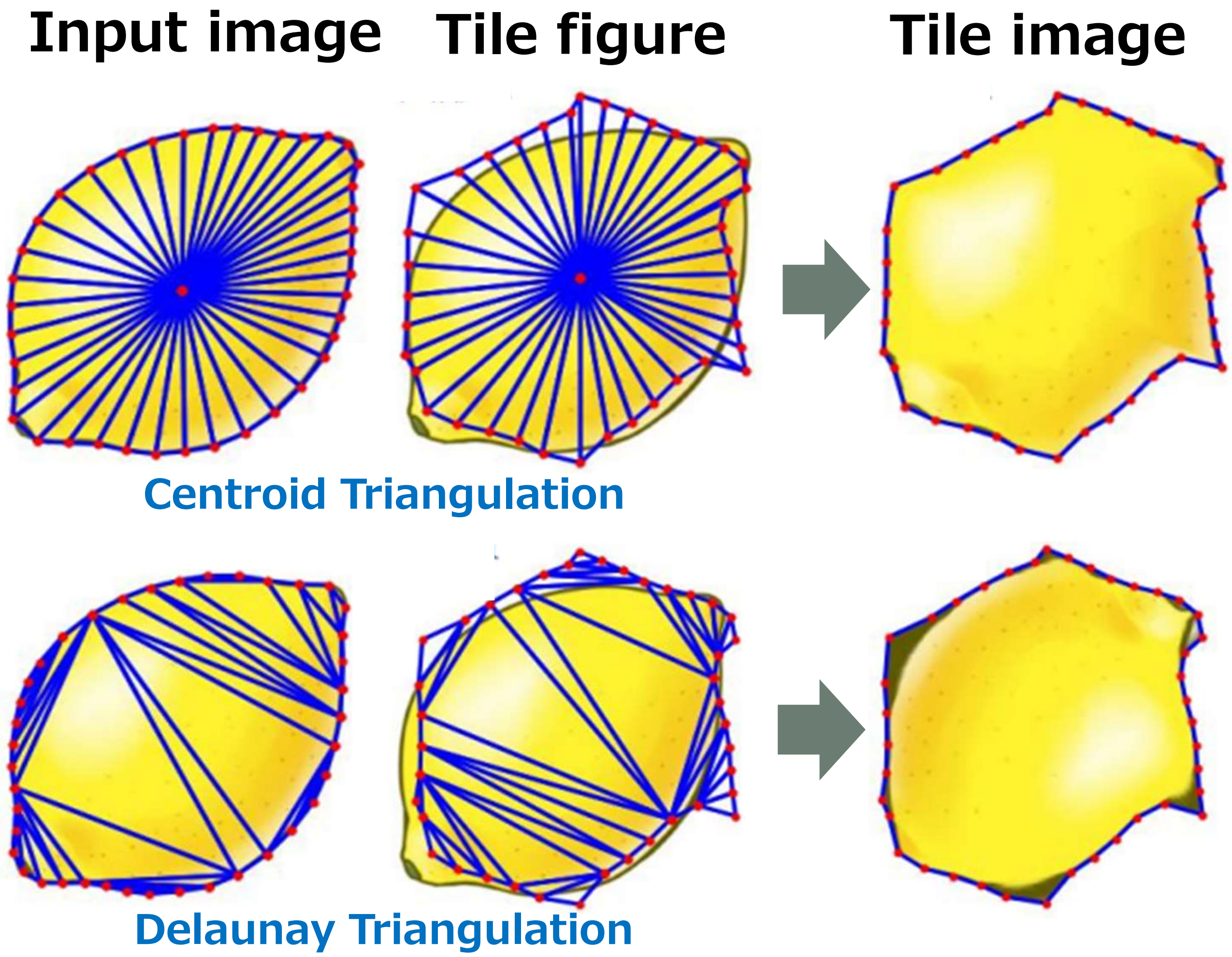
Proposed Method: Boundary Constraints

① The boundary line must be straight.

② The length ratio of the boundary figure must be equal.



Proposed Method: Tile Image



Centroid Triangulation
Divide the figure into triangles by connecting the centroid to each vertex.

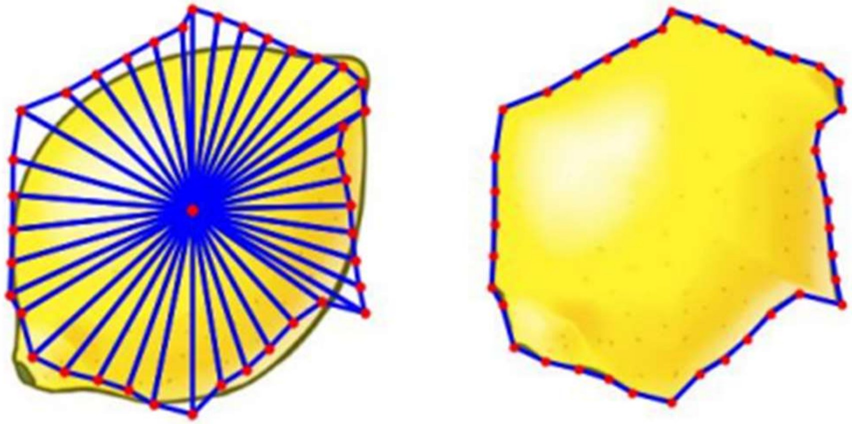
Delaunay Triangulation
Divide the figure into triangles in a way that the smallest angle in each triangle is as large as possible.

Object Transformation
Apply Affine transformation to each triangle.

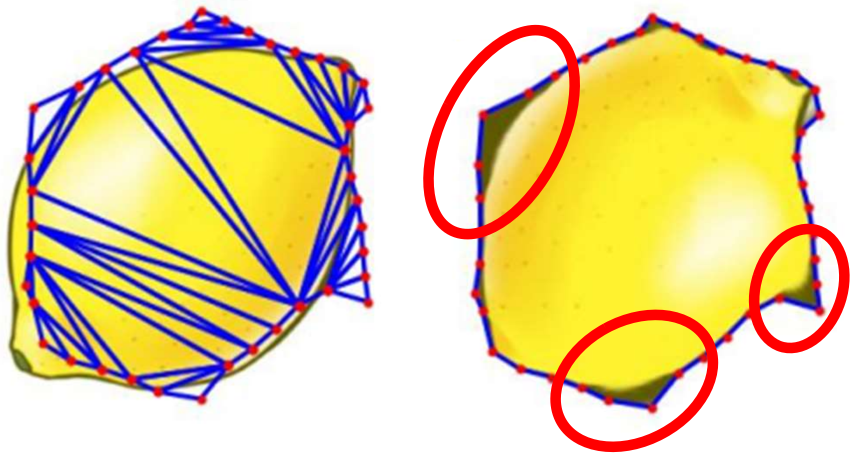
Experiment on Triangulation

Centroid triangulation is effective for simple objects.

Centroid triangulation



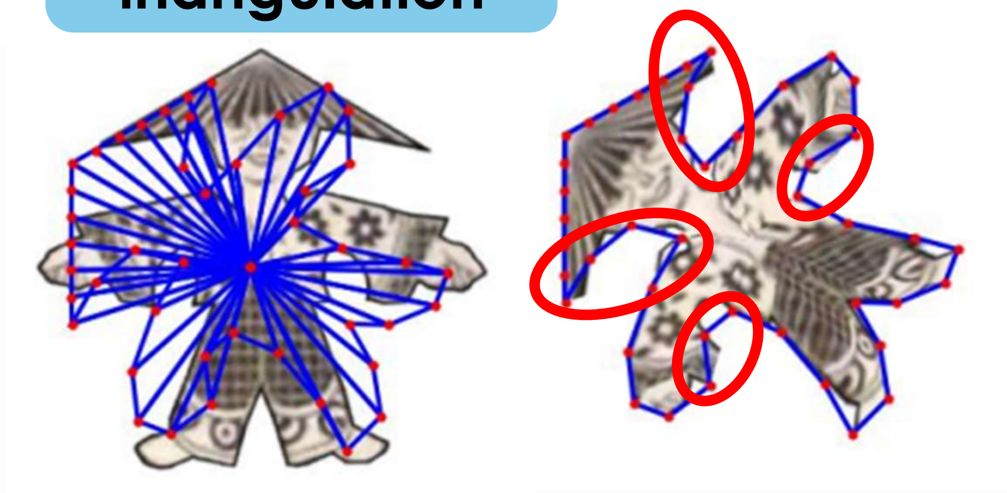
Delaunay triangulation



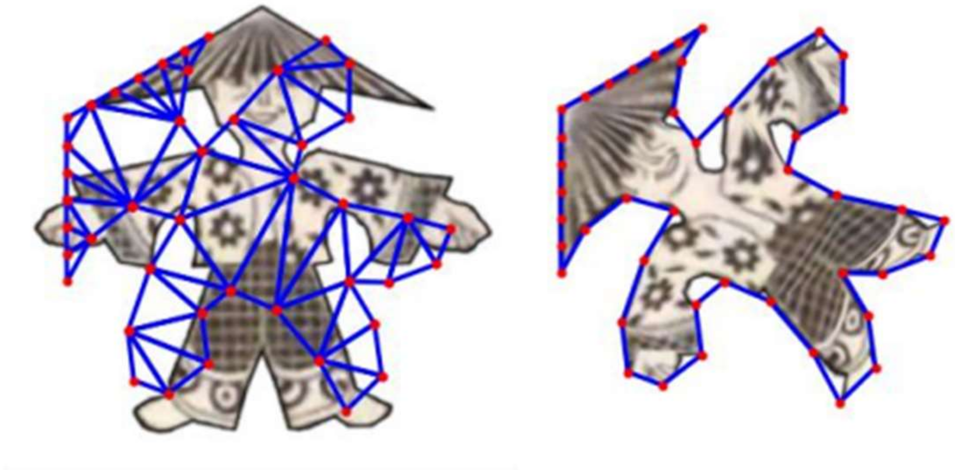
Experiment on Triangulation

Delaunay triangulation is effective for complex objects.

Centroid triangulation



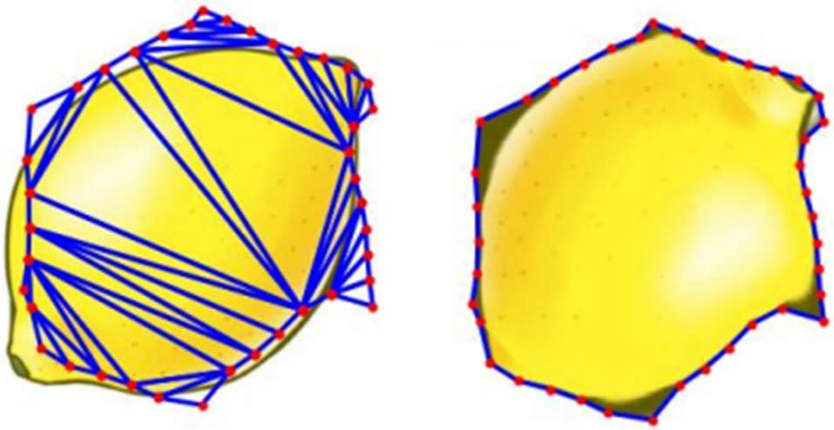
Delaunay triangulation



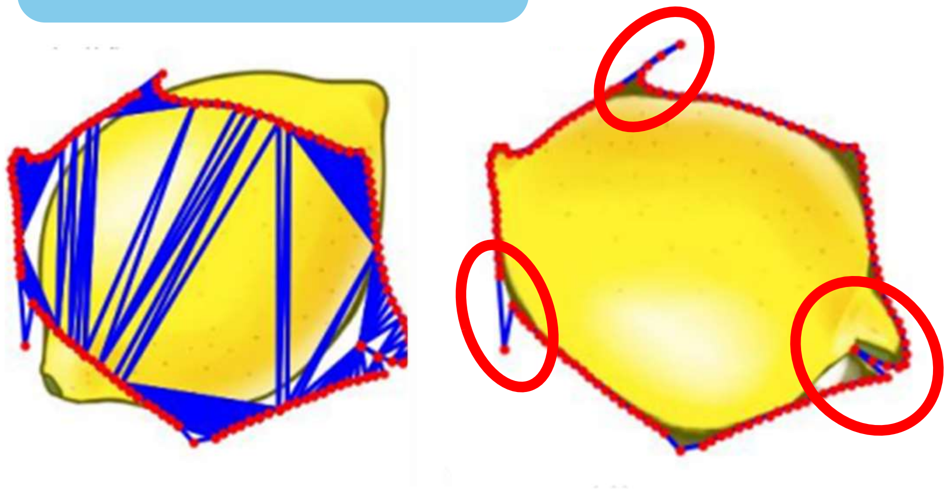
Experiment on the Number of Vertices

A larger number of vertices is ineffective for simple objects.

Number of vertices 36

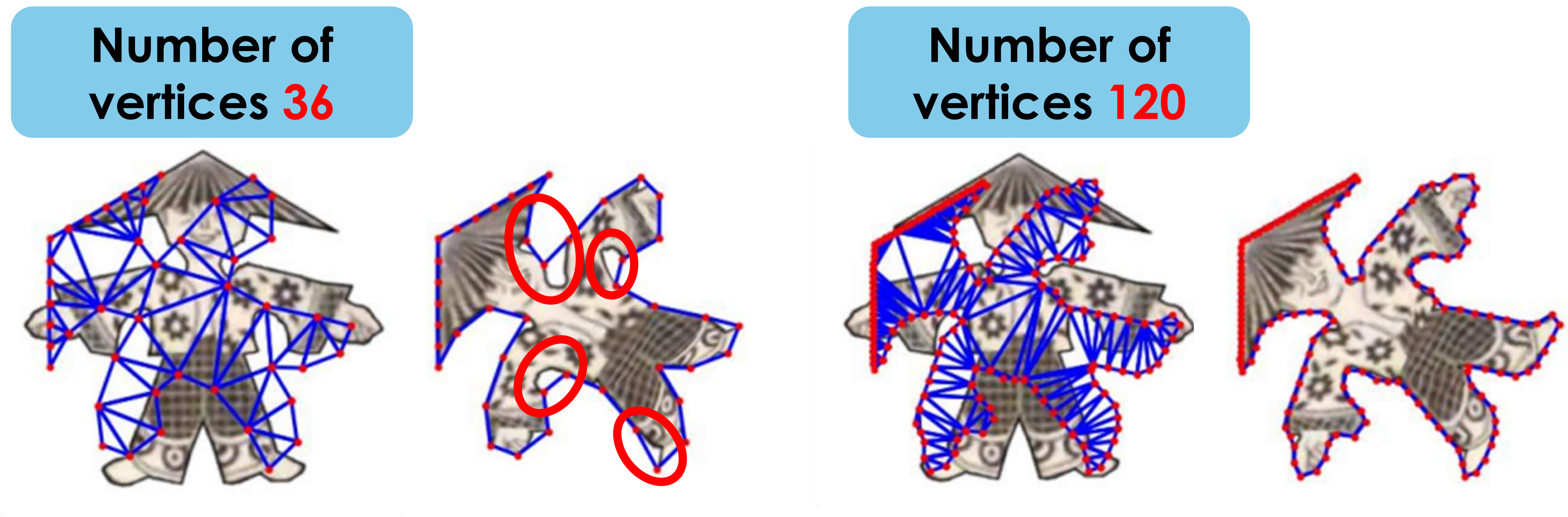


Number of vertices 120



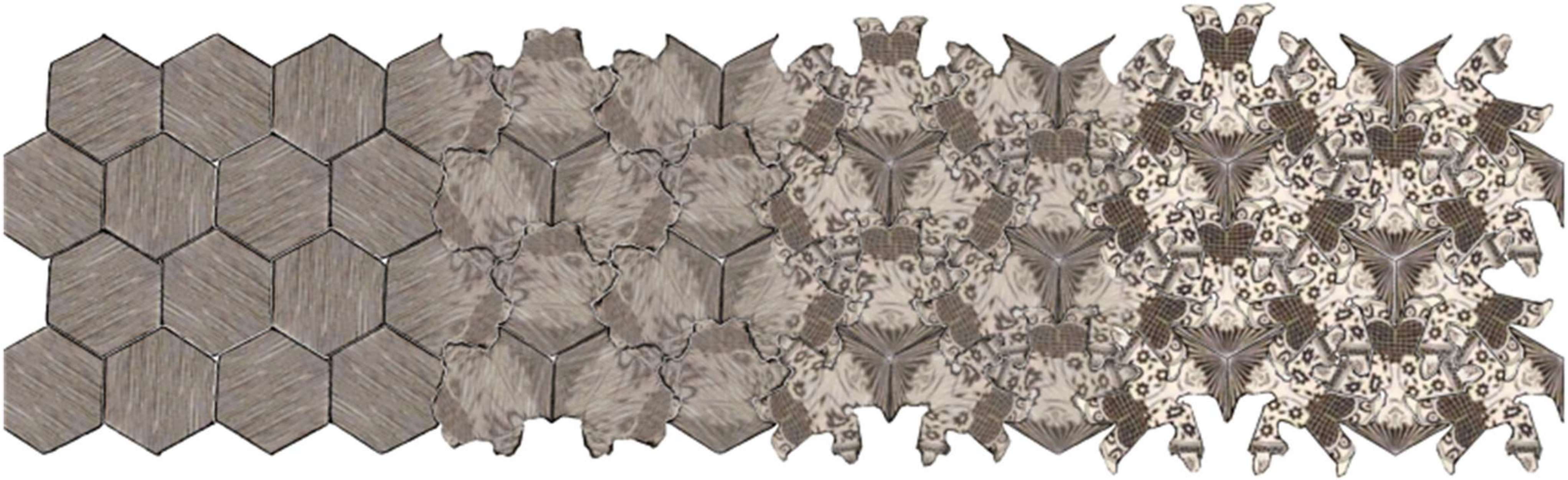
Experiment on the Number of Vertices

A larger number of vertices is effective for complex objects.



Reproducing Escher's Metamorphosis (1)

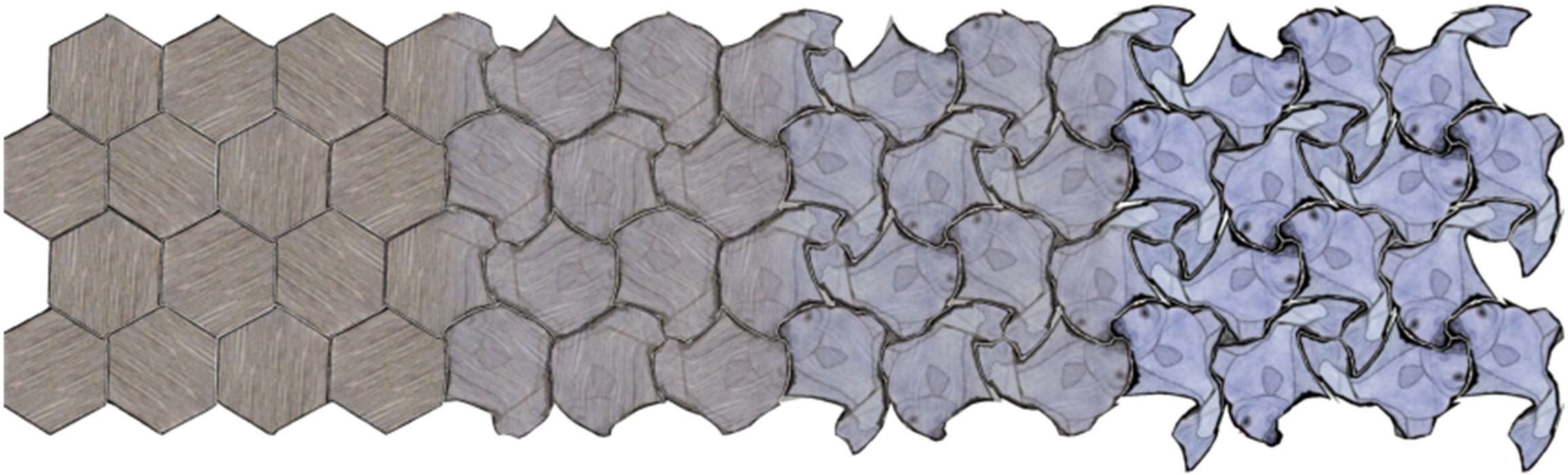
input: **Hexagon and Human**, **number of vertices**: **120**
triangulation: **Delaunay triangulation**



Reproducing Escher's Metamorphosis (2)

input: Hexagon and Fish, **number of vertices**: 36

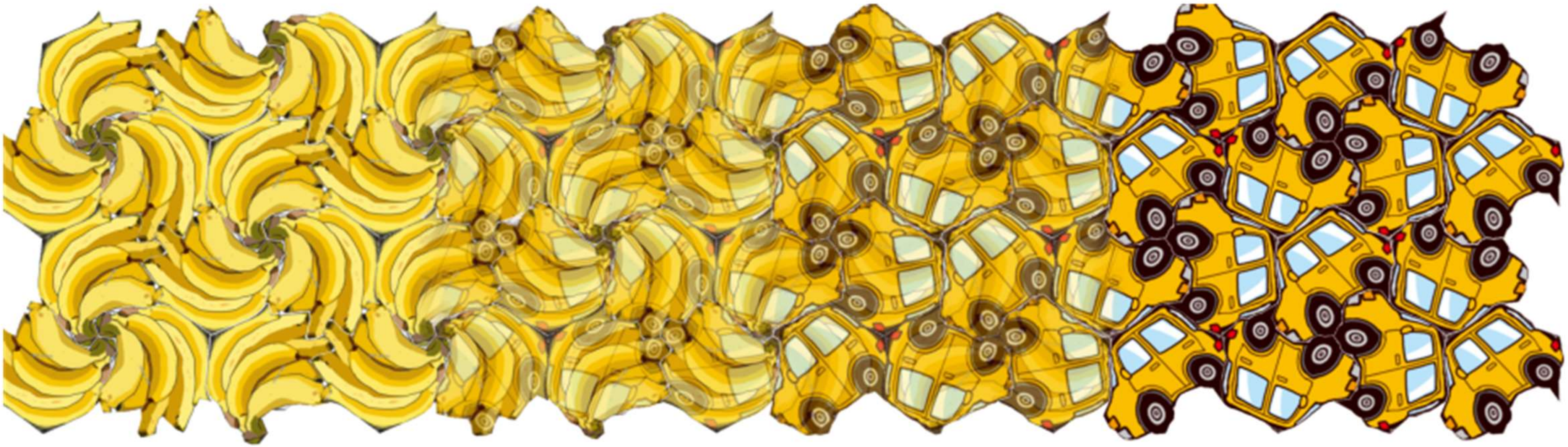
triangulation: Delaunay triangulation



Generating Original Metamorphosis (1)

input: Banana and Yellow Car,   **number of vertices**: 36

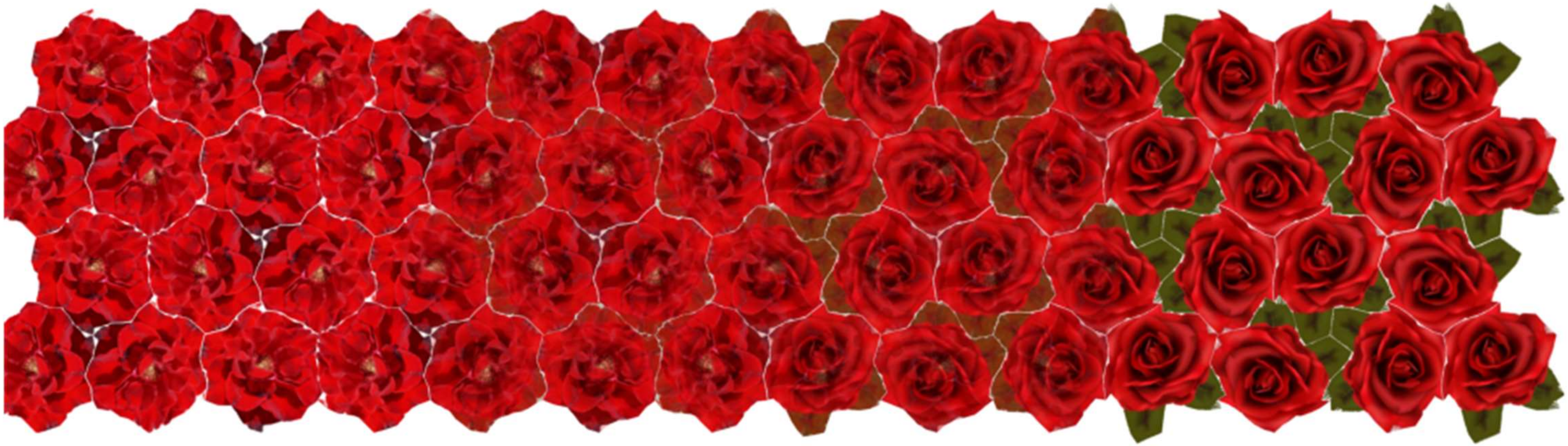
triangulation: Delaunay triangulation



Generating Original Metamorphosis (2)

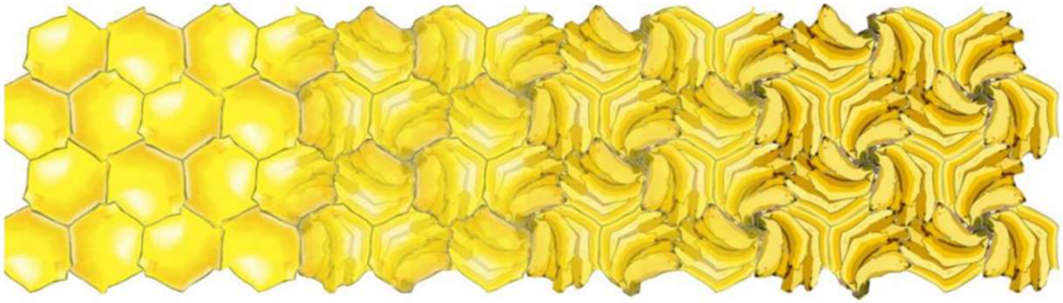
input: Peony and Rose,   **number of vertices**: 36

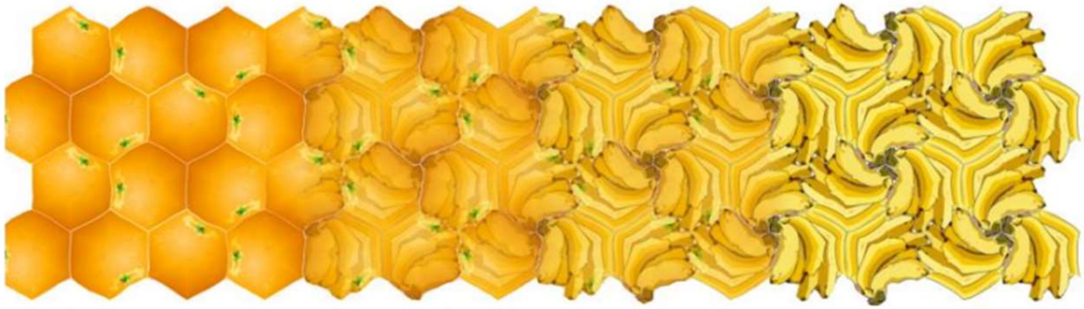
triangulation: Delaunay triangulation

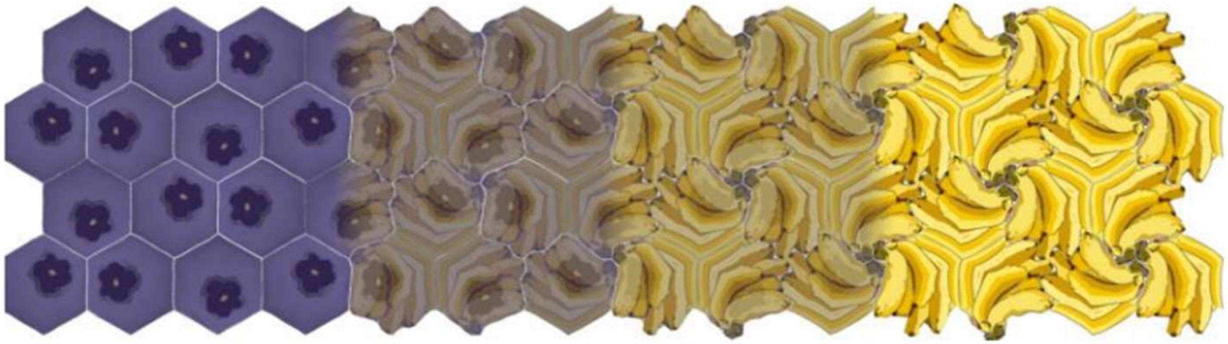


Experiment on Color Changes

Color difference in the input images results in unnatural color transition.

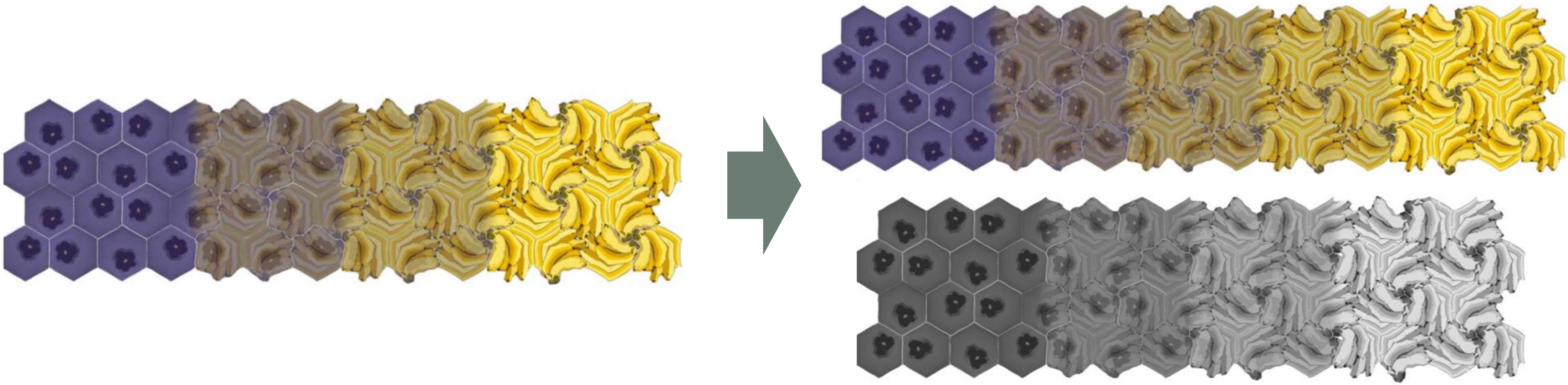






Experiment on Color Changes

- Increasing the number of intermediate image has no effect.
- Converting to grayscale has a slight effect.



Conclusion

- We built a system that **automatically generates morphing images** capturing the characteristics of Metamorphosis.
- The system successfully reproduces part of Escher's Metamorphosis and produces **original Metamorphosis**.
- Future challenge includes application of **machine learning** techniques to explore appropriate settings such as triangulation method and number of vertices.