Abduction, Unpredictability and Garden of Eden

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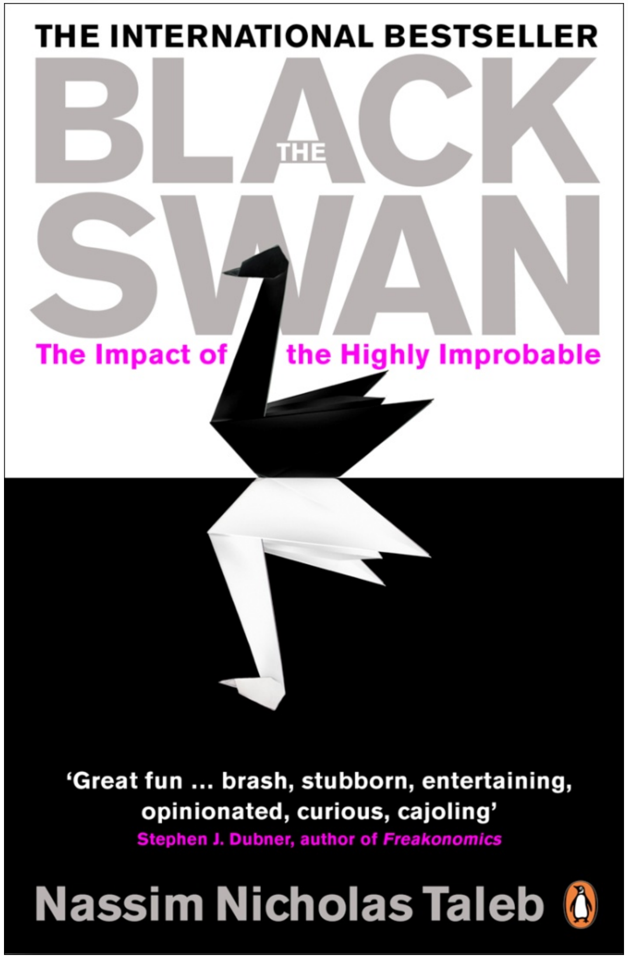
- A most powerful earthquake hit the north-east coast of Japan and triggered a massive tsunami.
- The quake and the tsunami caused explosions and meltdown of nuclear plants at Fukushima, which has raised great concern about the safety of nuclear power stations all over the world.
- Japanese experts said that such a powerful earthquake was ``**unthinkable**'' and the accident happened at the nuclear plant was ``**unexpected**''.



"The inability to predict outliers implies the inability to predict the course of history"

Contributions

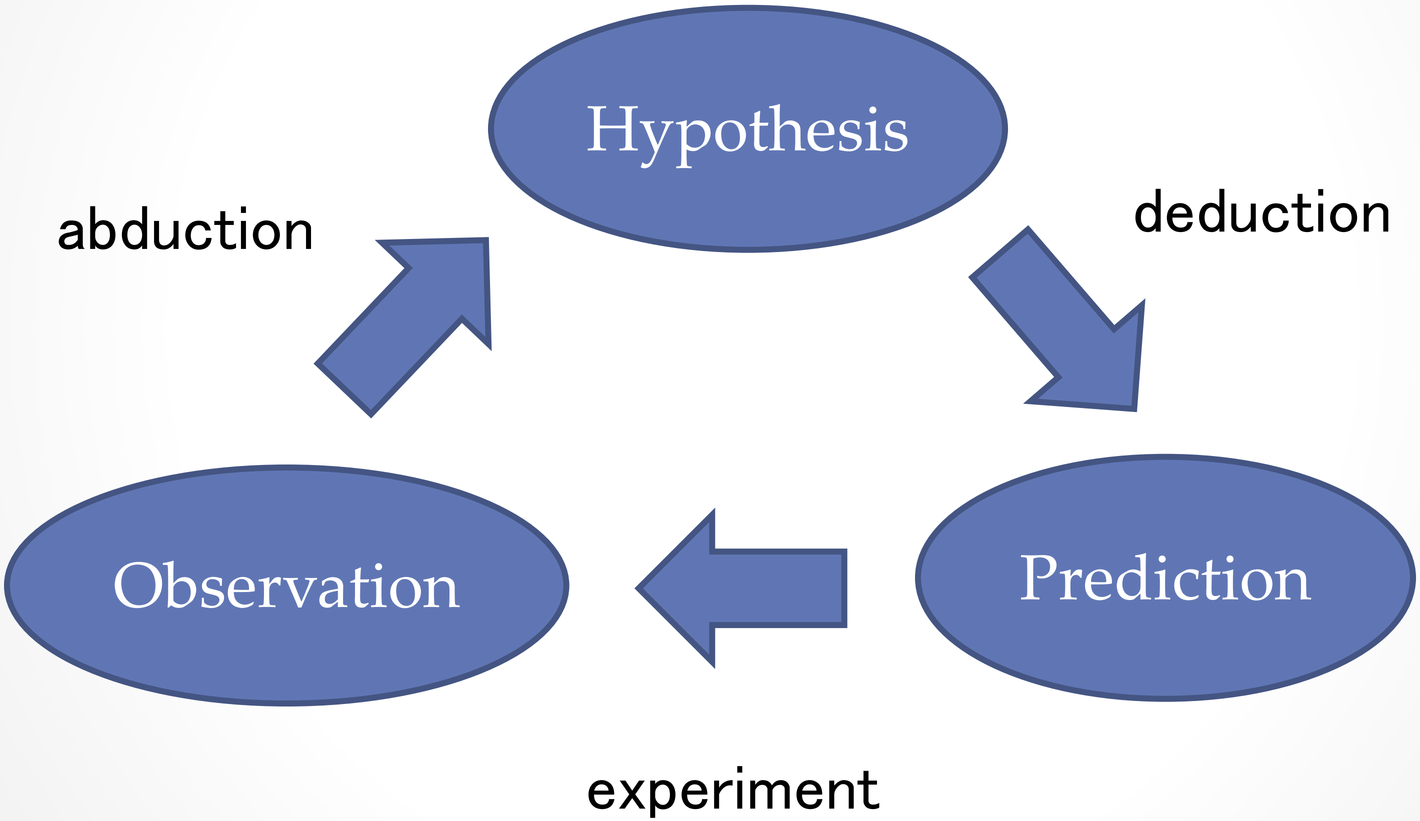
- Provide a formal account of “(**un**)**predictability**” based on **abduction**.
- Apply the notion of (un)predictability to the problem of identifying patterns in **cellular automata (CA)**.
- Provide computational methods of CA configurations in **logic programming**.

What is "(un)predictable"?

- if something is **predictable**, you know in advance that it will happen or what it will be like
- if something is **unpredictable** that cannot be predicted because it changes a lot or depends on too many different things

— ***Oxford Advanced Learner's Dictionary***

Hypothetico-Deductive Model



Abductive Framework

- A background theory **B**, an observation **O**, a hypothesis **H** and an event **E** are all (consistent) first-order formulas.
- An **abductive framework** is a pair <**B**, $\mathcal{H}$> where $\mathcal{H}$ is a set of formulas representing possible hypotheses.
- Each element in $\mathcal{H}$ is called an **abducible**.

Abduction and Prediction

- Given an abductive framework <**B**, $\mathcal{H}$> and an observation **O**, a hypothesis **H**$\in\mathcal{H}$ is an **explanation** of **O** under <**B**, $\mathcal{H}$> if

  1. $\mathbf{B} \wedge \mathbf{H} \models \mathbf{O}$,
  2. $\mathbf{B} \wedge \mathbf{H}$ is consistent.

- In this case, **O** is **explained** by **H** under <**B**, $\mathcal{H}$>.

- The hypothesis **H** may deduce new events under **B**. An event **E** is a **prediction** by **H** if $\mathbf{B} \wedge \mathbf{H} \models \mathbf{E}$.

(Un)predictability: Definition

- An event **E** is **predictable** by $\mathbf{H} \in \mathcal{H}$ under $<\mathbf{B}, \mathcal{H}>$ if

  1. $\mathbf{B} \wedge \mathbf{H} \models \mathbf{E}$,
  2. $\mathbf{B} \wedge \mathbf{H}$ is consistent.

- An event **E** is **predictable** under $<\mathbf{B}, \mathcal{H}>$
  if **E** is predictable by some $\mathbf{H} \in \mathcal{H}$ under $<\mathbf{B}, \mathcal{H}>$.

- An event **E** is **unpredictable** under $<\mathbf{B}, \mathcal{H}>$
  if **E** is not predictable by any $\mathbf{H} \in \mathcal{H}$ under $<\mathbf{B}, \mathcal{H}>$.

Explainability and Predictability

- Let <**B**, $\mathcal{H}$> be an abductive framework.
  An event **E** is predictable under <**B**, $\mathcal{H}$>
  iff **E** is explainable under <**B**, $\mathcal{H}$>.

Example

- Let <**B**, $\mathcal{H}$> be the abductive framework such that:

  B: ***p*** → ***r***, **q** → ¬ ***r***, ***s*** → ***t***.

  $\mathcal{H}$: ***p***, ***q***.

  In this case, ***p***, ***q***, ***r*** and ¬***r*** are predictable under <**B**, $\mathcal{H}$>, while **s** and **t** are unpredictable.

- The unpredictability is due to the incompleteness of hypothesis.

- Let <**B**, $\mathcal{F}$> be an abductive framework where $\mathcal{F}$ is the set of all formulas. Then, any event **E** is predictable under <**B**, $\mathcal{F}$> iff **B** ∧ **E** is consistent.

Formal Properties

Let <**B**, $\mathcal{H}$> be an abductive framework.

- If an event **E** is predictable by **H** under <**B**, $\mathcal{H}$> and an event ¬**E** is predictable by **H'** under <**B**, $\mathcal{H}$>, then **B** ∧ **H** ∧ **H'** is inconsistent.
- If **B** ⊨ ¬ **E,** then **E** is unpredictable under <**B**, $\mathcal{H}$>.
- In propositional abductive framework, the complexity of deciding whether an event is (un)predictable is Σ^p_2-complete.

Complex Systems

- A large-scale complex, organized, and adaptive behavior can emerge from simple interactions among myriad individuals.
- Complex systems are applied to a broad range of biological, technological, and social phenomena.
- Dynamics of a system are complex in its nature, and the behavior is generally **unpredictable**.

A New Kind of Science

Stephen Wolfram (2002)

- *"For our every day experience has led us to expect that an object that looks complicated must have been constructed in a complicated way … But … at least sometimes such an assumption can be completely wrong*".
- He observes this phenomenon in **cellular automata (CAs)** in which complex behaviors of a system emerge from a simple initial condition and a simple transition rule.
- *"It is this basic phenomenon that is ultimately responsible for most of the complexity that we see in nature*."

Cellular Automata (CA)

- A CA consists of a regular grid of **cells**.
- A cell has a finite number of possible **states**.
- The state of each cell changes synchronously in discrete time steps according to local and identical **transition rules**.
- The state of a cell in the next time step is determined by its current state and the states of its surrounding cells (**neighborhood**).
- The collection of all cellular states in the grid at some time step is called a **configuration**.

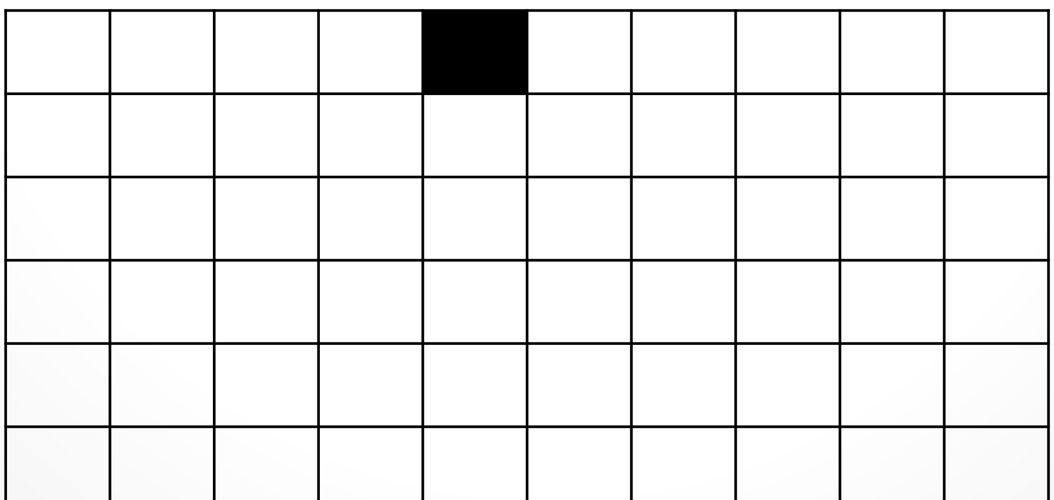
1-dimensional CA

current pattern	111	110	101	100	011	010	001	000
new state for center cell	0	0	0	1	1	1	1	0

Wolfram's Rule 30

0:white　1:black

t=0



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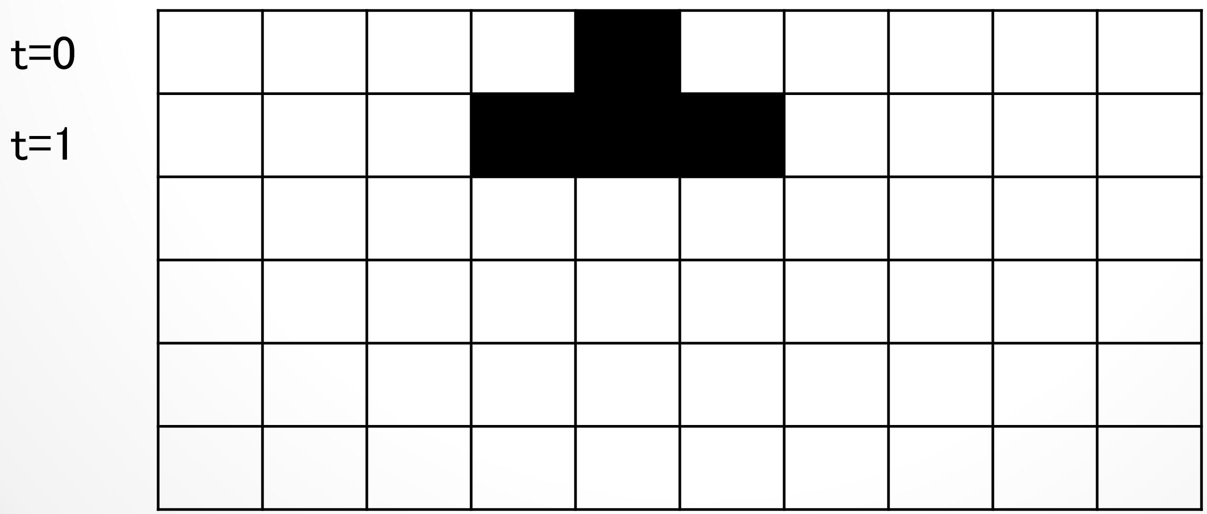

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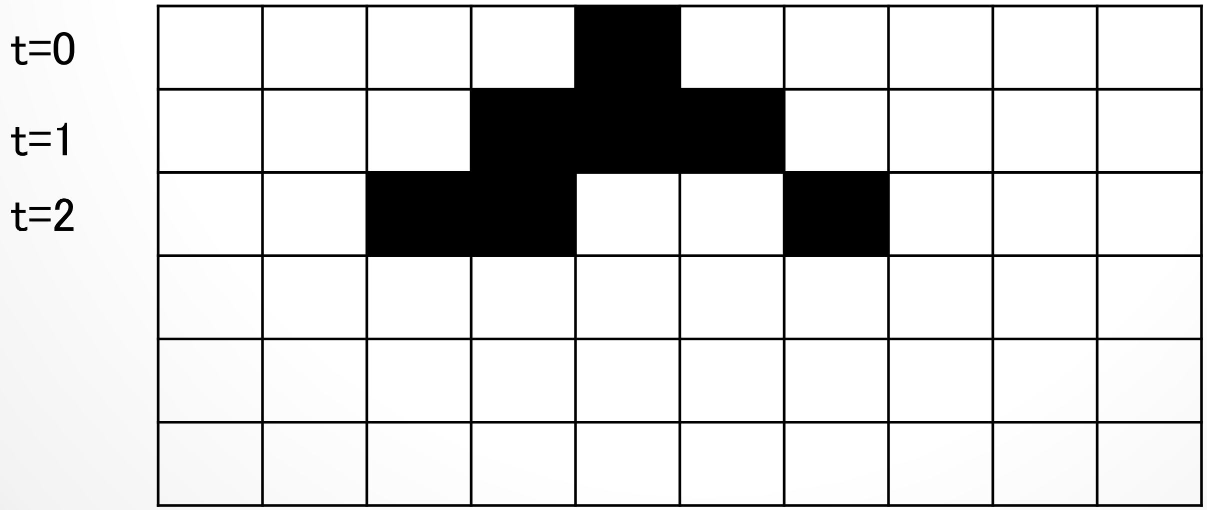

1-dimensional CA

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new state for center cell	0	0	0	1	1	1	1	0

Wolfram's Rule 30

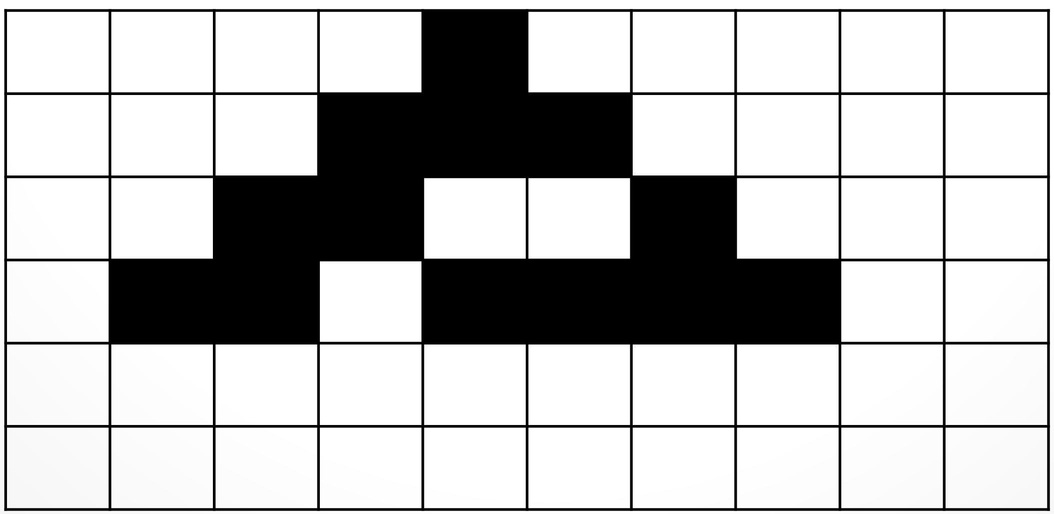
0:white 1:black

t=0

t=1

t=2

t=3



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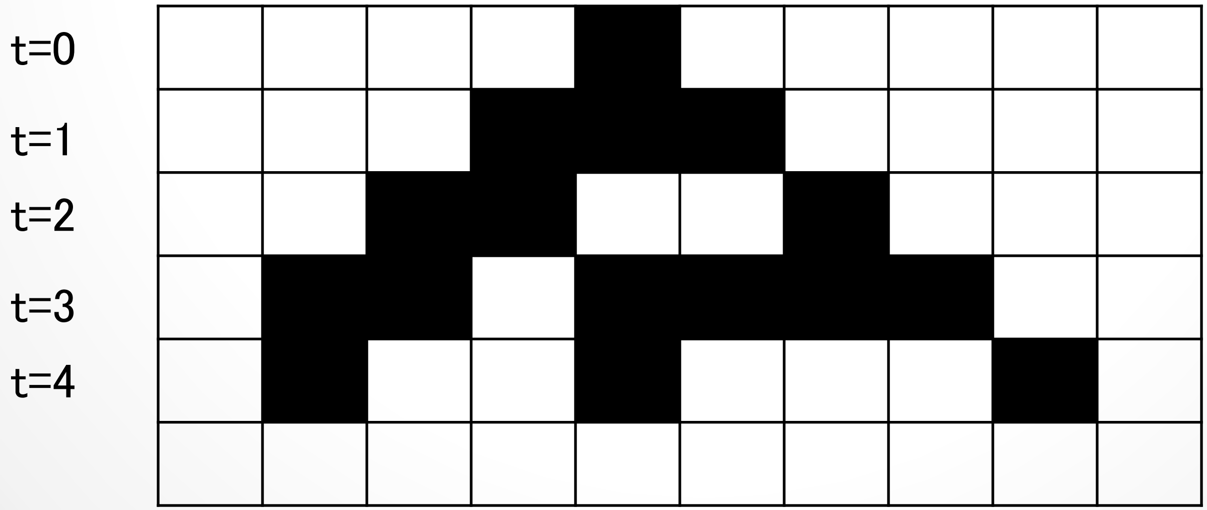

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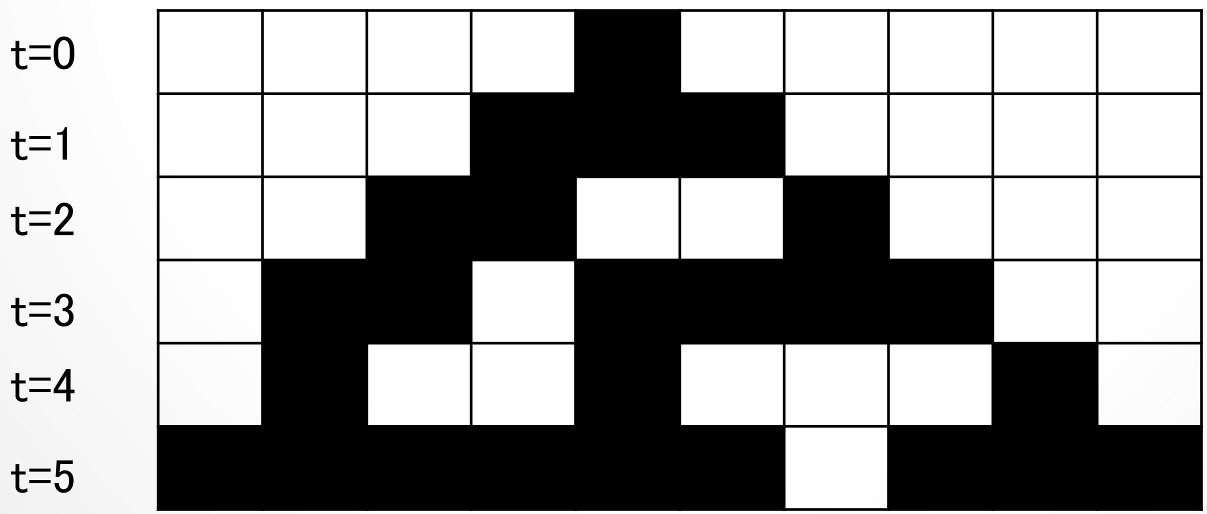
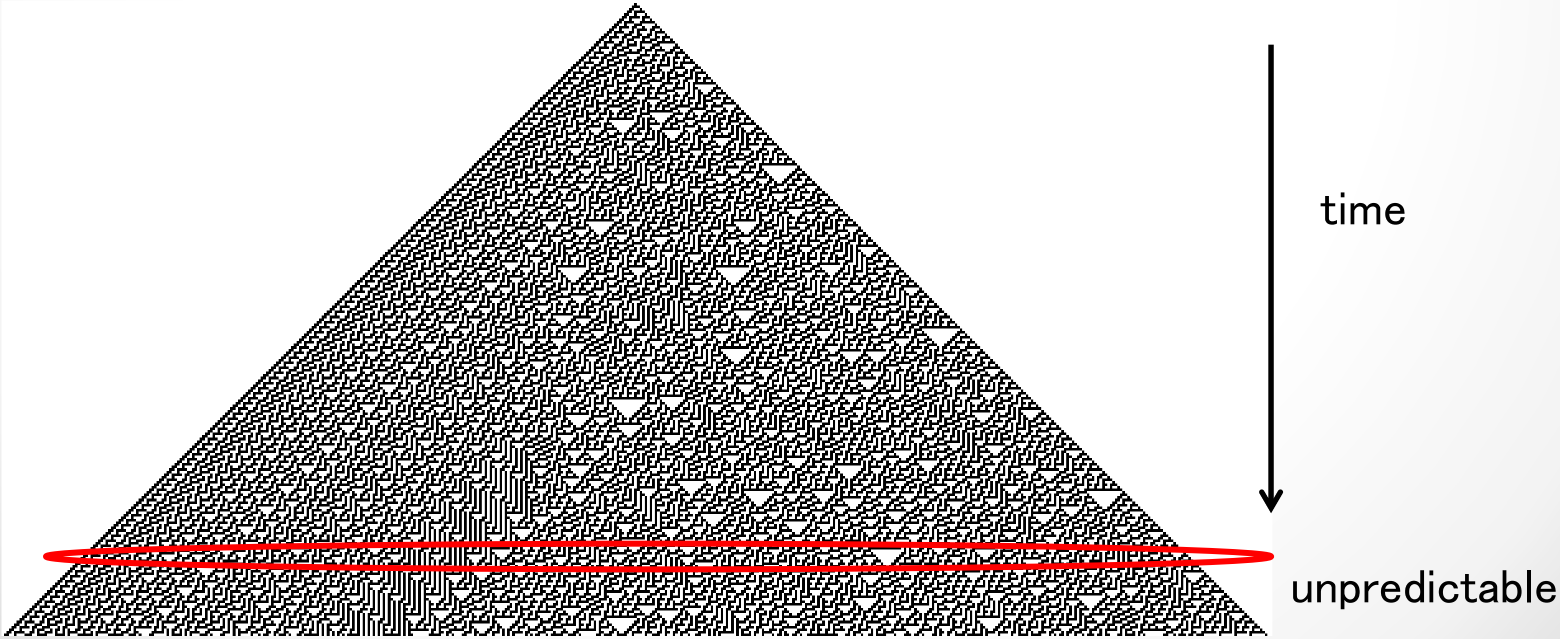


Unpredictability in CA

- Evolving patterns depend strongly on the initial configuration and a rule used. (2^8=256 possible rules)
- Future states are **unpredictable** unless doing a state by state computation.



Representing CA in Abductive Framework

- Represent transition rules as a background knowledge **B**.

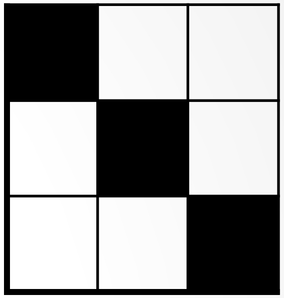
  ex. Rule 30: $x_i^{t+1} = x_{i-1}^t \oplus (x_i^t \vee x_{i+1}^t)$

x_{i-1}^t	x_i^t	x_{i+1}^t	x_i^{t+1}
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

- Every possible initial configuration is put in a hypotheses space $\mathcal{H}$.

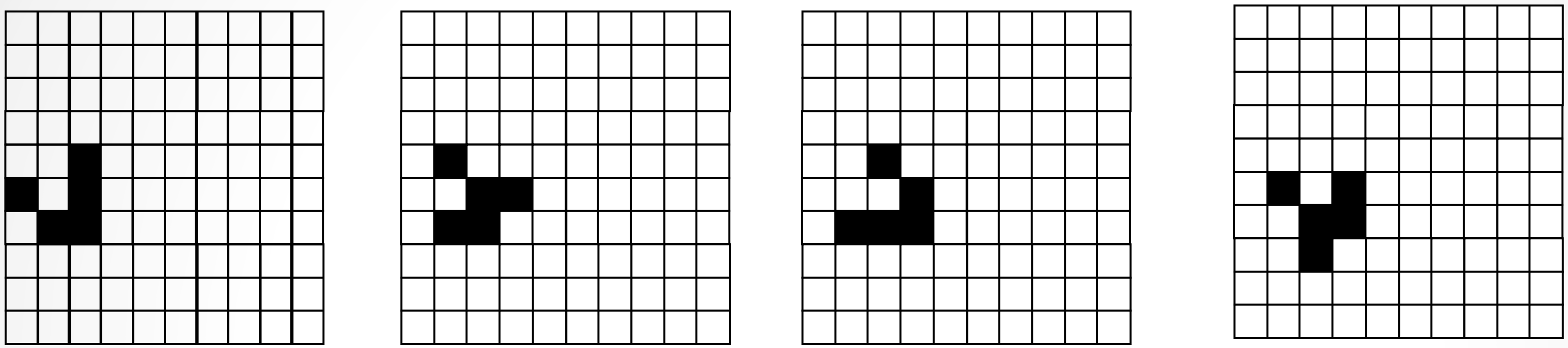
- Then, a configuration **E** is produced from an initial configuration **H** using a transition rule of **B** iff **E** is predictable by **H** $\in \mathcal{H}$ under <**B**, $\mathcal{H}$>.

Game of Life (LIFE)

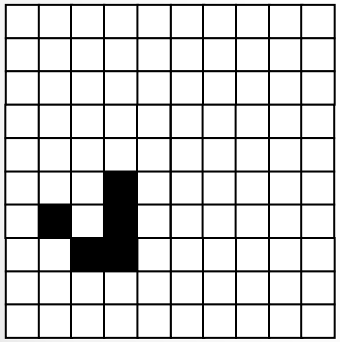


- 2-dimensional 2-state CA.
- The state of a cell is either black (alive) or white (dead).
- A neighborhood consists of a central cell and the nearest 8 surrounding cells.
- Cellular states change according to the CA-rules:
  - o Any live cell with 2 or 3 live neighbors stays alive at the next time step; otherwise, the cell dies of loneliness or overcrowding at the next time step.
  - o Any dead cell with exactly 3 live neighbors will become alive.

The “Glider” Pattern in LIFE



t=0 t=1 t=2 t=3



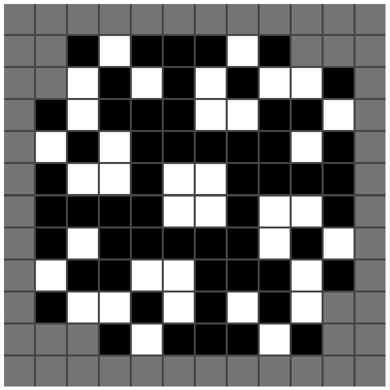
t=4

....

The pattern at t=0 moves across the grid and appears at t=4.

Garden of Eden

- A **Garden of Eden (GOE)** is a configuration with no predecessor. That is, a GOE cannot be reached from any initial configuration.
- An **orphan** is a finite pattern that has no redecessor regardless of the states of the neighbors.
  Each GOE contains at least one such an orphan.
- Rule 30 of 1-dimensional CA has no GOE.
- In LIFE, the smallest known orphan was found in December 2011, which has 56 living cells and fits in an 10 x 10 square.



GOE and Unpredictability

- A GOE is considered a configuration that is unpredictable by any predecessor under a transition rule.
- Let **B** be a background theory specifying a transition rule of a CA and $\mathcal{H}$ is the set of possible initial configurations. Then, a configuration **E** is a GOE iff **E** is unpredictable under <**B**, $\mathcal{H}$>.
- The results provide connections between (un)predictability in formal logic and the corresponding notion in CAs.

Computation in Logic Programming

- An abductive framework for CAs is realized by (**abductive**) **logic programming**.
- We have implemented LIFE and verified that the GOE of 10x10 is not generated by one step of deduction from any configuration of a 12x12 grid.
- It is known that there are no GOE that are 6x6 cells wide or smaller. It is still open whether there is any GOE between 7x7 and 9x9 grids.