The PLP System

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Preferential ASP

PLP
Sakama & Inoue
1996 prop.
2003 impl.

PLP
Brewka & Eiter
1998 prop.
2003 impl.

OLP
Delgrande, Schaub & Tompits
1998 prop.
2003 impl.

LPOD
Brewka, Niemela & Syrjanen
2002 prop. / impl.

etc.

Prioritized Logic Programs

- **PLP (*P*, *Φ*)**

  Sakama and Inoue [JICSLP-96, AIJ 2000]

  - Explicit representation of priorities in ASP
  - Realize various forms of nonmonotonic reasoning
  - Semantics: **preferred answer sets**

- **Implementation of PLP**
  - Sakama and Inoue's naïve procedure [AIJ 2000]
    - Applicable to a limited classes of LP
  - Wakaki, Inoue, Sakama & Inoue [LPAR'03]
    - Meta-programming in ASP

Wakaki et al.'s Approach

- Construct a logic program T[P, Φ, S] from a PLP (P, Φ) and an answer set S of P.
- The answer sets of T[P, Φ, S] are those answer sets of P which are strictly preferable to S.
- The emptiness of the answer sets of T[P, Φ, S] implies that S is a preferred answer set of (P, Φ).
- A similar technique can also be applied to computing ***prioritized circumscription*** in ASP [Wakaki & Inoue, ICLP 2004].

Prioritized Logic Programs

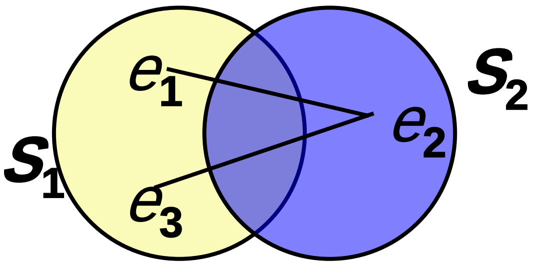
- Prioritized logic program
  - $\langle P, \Phi \rangle$
    - P : General extended disjunctive program
    - Φ : Set of priorities on ***literals***
- Priority relation $\leq$: reflexive and transitive
  - $e_1 \leq e_2$: "e_2 has a priority over e_1"
  - $e_1 \prec e_2$: $(e_1 \leq e_2)$ and $\neg(e_2 \leq e_1)$

Preferred Answer Set Semantics

- Priority relation $\sqsubseteq$: reflexive and transitive
  - $S_1 \sqsubseteq S_2$: $\exists\, e_2 \in (S_2 \setminus S_1)$,

    (i) $\exists\, e_1 \in (S_1 \setminus S_2)$ s.t. $e_1 \leq e_2$

    (ii) $\neg\exists\, e_3 \in (S_1 \setminus S_2)$ s.t. $e_2 \prec e_3$



  - $S \in$ **AS**(P) is a **preferred answer set** of P if

    $\forall\, S' \in$ **AS**(P). $S \sqsubseteq S' \rightarrow S' \sqsubseteq S$.

Meta-programming for Preference Translation

- The priorities Φ and a GEDP P are represented in the program $T[P, \Phi, S]$ s.t. $c \leq d \in \Phi$ iff $\leq(c_t, d_t)$ where c_t, d_t are terms representing literals c, d.

- $L \in S$ is **renamed** by a newly introduced atom L^* to compare the given answer set S with another answer set S' in an answer set of $T[P, \Phi, S]$.

- For a term c_t representing a literal c and its renamed term c^*, $m_1(c_t)$ and $m_2(c_t)$ represents $c \in S$ and $c \in S'$, respectively.

$$T[P, \Phi, S] \stackrel{\text{def}}{=} P \cup \Gamma \cup \Pi$$

Γ :

1. $L^* \leftarrow,$ for any $L \in S$

2. $\precsim(a_t, b_t) \leftarrow,$ for any $a \precsim b \in \Phi$

3. $m_1(L_t) \leftarrow L^*,$
   $m_2(L_t) \leftarrow L,$

   for any $L \in Lit_P$

Π :

4. $\preceq(x, x) \leftarrow$

5. $\preceq(x, z) \leftarrow \preceq(x, y), \preceq(y, z).$

6. $\prec(x, y) \leftarrow \preceq(x, y), \; not \; \preceq(y, x).$

7. $gr_1(x,y) \leftarrow m_1(x), \preceq(x,y), m_2(y),$
$not \; m_2(x), not \; m_1(y).$

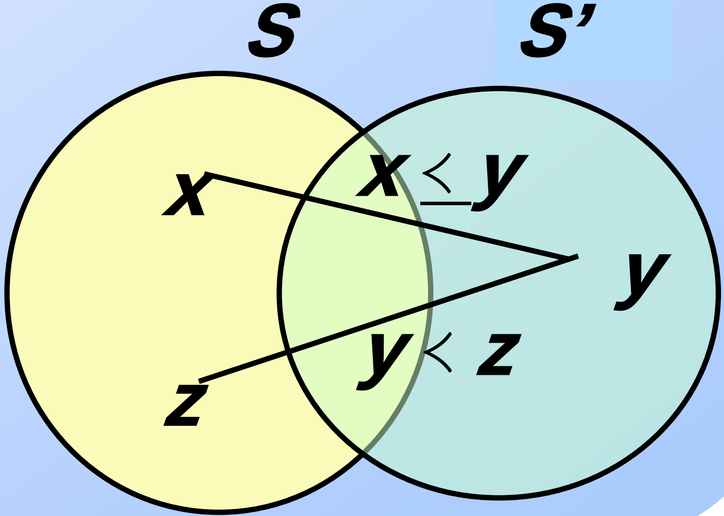
8. $gr_2(y,z) \leftarrow m_2(y), \prec(y,z), m_1(z),$
$not \; m_1(y), not \; m_2(z).$

9. $attacked(y) \leftarrow gr_2(y,z)$

10. $defeated(x) \leftarrow gr_1(x,y),$
$not \; attacked(y).$

11. $better \leftarrow defeated(x).$

12. $\leftarrow not \; better.$



Let $T[P, \Phi, S]$ be the GEDP constructed from a PLP (P, Φ) and an answer set S of P.

Theorem: (Soundness/Completeness)

$T[P, \Phi, S]$ *has an answer set* E *if and only if there is another answer set* S' *of* P *such that* $S \sqsubseteq S'$ *and* $S' = E \cap Lit_P$.

Corollary:

$T[P, \Phi, S]$ *is inconsistent if and only if* S *is a strictly preferred answer set of* (P, Φ).

Procedure to Compute the Preferred Answer Sets

***CompPAS* (P, Φ, Δ)**

Input: a PLP (P, Φ)

Output: the set Δ of all preferred answer sets of (P, Φ)

Step1: Compute the set AS of all answer sets of P.

Step2: If Φ = {}, then $\Delta := AS$, return Δ;

Else, let $\Omega := \{ s_i \mid 1 \leqq i \leqq |AS| \}$ in which each answer set $S \in AS$ is assigned a unique ID $s_i \in \Omega$.

Step3: Let $\Sigma :=$ {}; For each answer set $S \in AS$,

if $T[P, \Phi, S]$ is consistent, then for each its answer set E, put $\Sigma := \Sigma \cup \{ \sqsubseteq(s, s') \leftarrow \}$, where $S' = E \cap Lit_p$ and $s, s' \in \Omega$ are the answer set IDs for S and S', respectively.

Step4: Compute an answer set ***U*** of the logic program:

$$\Sigma \cup \{\ \mathbf{as}(\mathbf{s}) \leftarrow \ |\ \mathbf{s} \in \Omega\ \} \cup \Psi$$

Ψ **:**

$$\sqsubseteq(x, x) \leftarrow \mathbf{as}(x).$$

$$\sqsubseteq(x, z) \leftarrow \sqsubseteq(x, y),\ \sqsubseteq(y, z).$$

$$\sqsubset(x, y) \leftarrow \sqsubseteq(x, y),\ \textbf{\textit{not}}\ \sqsubseteq(y, x).$$

$$\mathbf{worse}(x) \leftarrow \sqsubset(x, y).$$

$$\mathbf{p\text{-}as}(x) \leftarrow \mathbf{as}(x),\ \textbf{\textit{not}}\ \mathbf{worse}(x).$$

Step5: Return

Δ := { $S \in \boldsymbol{AS}$ | The answer set ID ***s*** of ***S*** satisfies

p-as (*s*) $\in \boldsymbol{U}$ }

Example

PLP (P, Φ)

P: $p \mid q \leftarrow$

$q \mid r \leftarrow$

Φ: $\{\, p \preceq q \,,\ q \preceq r \,\}$

Step 1: Answer sets of P:

$S_1 = \{\, p\,,\, r \,\}$, $S_2 = \{\, q \,\}$

Step 3: $T[P, \Phi, S_1] = P \cup \Gamma_1 \cup \Pi \Rightarrow$ ***inconsistent***

Γ_1: $p^* \leftarrow$, $r^* \leftarrow$, $\preceq(p_t, q_t) \leftarrow$, $\preceq(q_t, r_t) \leftarrow$,

$m_1(p_t) \leftarrow p^*$, $m_1(q_t) \leftarrow q^*$, $m_1(r_t) \leftarrow r^*$,

$m_1(np_t) \leftarrow \neg p^*$, $m_1(nq_t) \leftarrow \neg q^*$, $m_1(nr_t) \leftarrow \neg r^*$,

$m_2(p_t) \leftarrow p$, $m_2(q_t) \leftarrow q$, $m_2(r_t) \leftarrow r$,

$m_2(np_t) \leftarrow \neg p$, $m_2(nq_t) \leftarrow \neg q$, $m_2(nr_t) \leftarrow \neg r$.

- $T[P, \Phi, S_2] \Rightarrow S_2 \sqsubseteq S_1$ S_1: a preferred AS

Legal Problem (Gorden, 1993)

***P*: posses ← , ship ← , ¬filstate ← ,**
perfected* ← *posses, not ab1, **(*UCC*)**
¬*perfected* ← *ship,* ¬*filstate, not ab2,* **(*SMA*)**
ab1* | *not ab1, ab2* | *not ab2,* ← *ab1,ab2,
ucc* ← *not ab1, sma* ← *not ab2.

Answer sets of ***P***:

S_1={ *perfected, posses, ship,*¬*filstate, ab2, ucc* }

S_2={ ¬*perfected, posses, ship,*¬*filstate, ab1, sma* }

Conflict between ***UCC*** and ***SMA***

The principle of *Lex Posterior* gives precedence to ***newer*** laws, while **the principle of *Lex Superior*** gives precedence to laws supported by the ***higher authority***.

- ***UCC*** is newer than ***SMA***.
- ***SMA*** has higher authority than ***UCC***.

$$
\begin{aligned}
\Phi_1 :\ & \textit{moreRecent}(ucc_t, sma_t) \leftarrow, \\
& \textit{fed}(sma_t) \leftarrow, \quad \textit{state}(ucc_t) \leftarrow, \\
& \textit{lp}(Y,X) \leftarrow \textit{moreRecent}(X,Y), \\
& \textit{ls}(Y,X) \leftarrow \textit{fed}(X),\ \textit{state}(Y), \\
& \preceq(Y,X) \leftarrow \textit{lp}(Y,X),\ \textit{not}\ \textit{conf}_1(X,Y), \qquad (LP) \\
& \preceq(Y,X) \leftarrow \textit{ls}(Y,X),\ \textit{not}\ \textit{conf}_1(X,Y), \qquad (LS)
\end{aligned}
$$

$$T[\,P, \Phi_1, S_1\,] \Rightarrow S_1 \sqsubseteq S_2$$

$$T[\,P, \Phi_1, S_2\,] \Rightarrow S_2 \sqsubseteq S_1$$

S_1, S_2 : preferred ASs

Conflict between ***LP*** and ***LS***

Meta-priority:

$LexPosterior(X,Y) \preceq LexSuperior(U,V)$

PLP (P, Φ_2):

$\Phi_2 = \Phi_1 \cup$
$\{ conf_1(Y, X) \leftarrow lp(X, Y), ls(Y, X), not\ conf_2(X, Y) \}$

$T[P, \Phi_1, S_2] \Rightarrow$ **inconsistent**

Only S_2 is a preferred answer set of (P, Φ_2).

$S_1 = \{$ *perfected*, *posses*, *ship*, $\neg$*filstate*, *ab2*, *ucc* $\}$

$S_2 = \{$ $\neg$*perfected*, *posses*, *ship*, $\neg$*filstate*, *ab1*, *sma* $\}$