Disjunctive Explanations

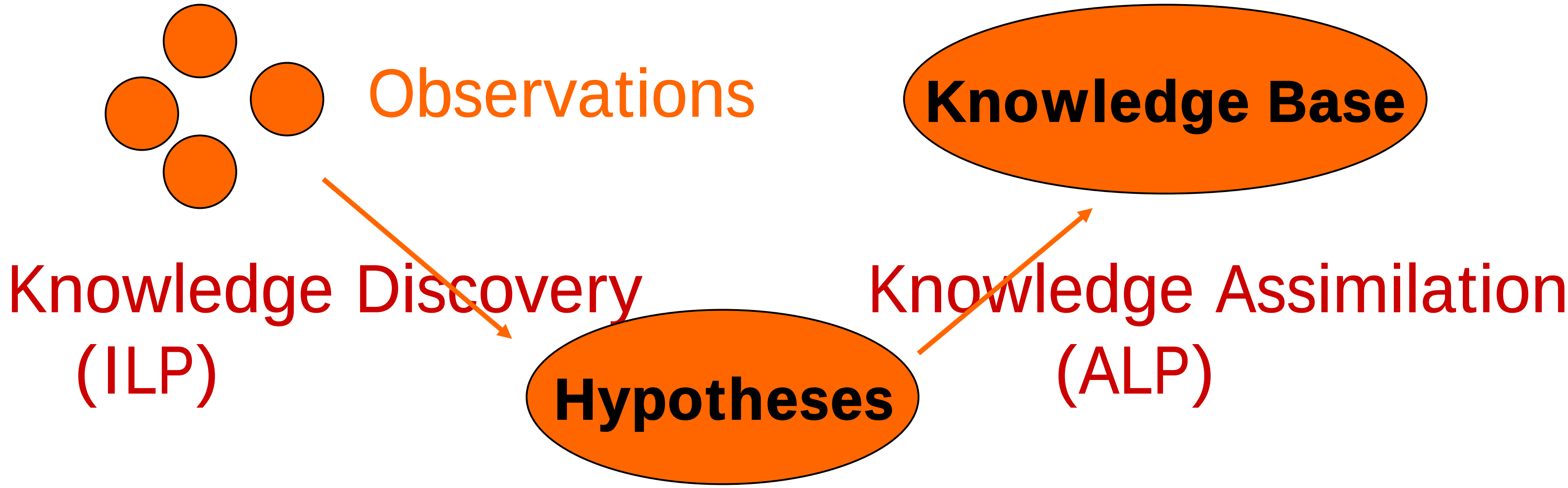
Katsumi Inoue
Kobe University
Chiaki Sakama
Wakayama University

2002.07.31 (**ICLP 2002)**

Knowledge Discovery and Abduction

Knowledge Discovery by Induction

Knowledge Assimilation by Abduction



(Normal) Abduction

- K : background theory
- G : **observation**
- Γ : set of **abducible** rules/literals

- $E \subseteq \Gamma$ is an **explanation** of G if:
  1. $K \cup E \models G$
  2. $K \cup E$ is consistent

- In general, many explanations exist.

Disjunctive Explanations

When $E1$ and $E2$ are explanations,

$E1 \vee E2$ is also an explanation, because

1. $K \cup \{E1 \vee E2\} \models G$
2. $K \cup \{E1 \vee E2\}$ is consistent.

- Usually, each Ei is minimal.
- $E1 \vee E2$ is **weaker** than each Ei.

How to Assimilate Explanations

- *Disjunctive explanation is assimilated.*
- **Merit:** The assimilated KB preserves the intended semantics from the collection of all possible updated KBs (<u>**this work**</u>).
- **Merit:** One current KB is only necessary at a time, still keeping every possible change in a single state.
- **DEmerit:** The assimilated KB is weaker than any possible single update.

Removing Disjunctions from KB

- To **delete** *p* in **view update** from

$$p \leftarrow a,$$
$$p \leftarrow b,$$
$$a \,;\, b\,,$$

  with *{a, b}* : **removable abducibles**,
  removing *{a}*, *{b}*, or *{a,b}* does not work.
- Instead, *{ a ; b }* should be removed.
- Not much work exists for this kind of *disjunctive update* in literature.

Minimal Change Problem

- To **insert** p in **view update** from

$$p \leftarrow \textbf{\textit{not}}\ a,$$
$$p \leftarrow \textbf{\textit{not}}\ b,$$
$$a\ ,$$
$$b\ ,$$

with $\{a, b\}$: (removable) abducibles,
removing $\{a\}$, $\{b\}$, or $\{a,b\}$ is too strong.

- In fact, we may remove either {a} or {b}, but how can we represent this indefiniteness?

Skeptical Abduction = Circumscription [Helft et al. 91]

- p is entailed by circumscribing $ab1$, $ab2$ in

$$\neg ab1 \supset p,$$
$$\neg ab2 \supset p,$$
$$ab1 \vee ab2 .$$

- This is verified by computing explanations of p: $\neg ab1$, $\neg ab2$, then combining these as

$$\neg ab1 \vee \neg ab2,$$

whose negation $ab1 \wedge ab2$ cannot be explained.

Disjunctive Hypotheses in Inductive Logic Programming

- Suppose we learned two rules for p as

$$C1(x) \supset p(x),$$
$$C2(x) \supset p(x).$$

- Taking the disjunction (lub) of these 2 rules corresponds to computing the **greatest specialization under implication**:

  $C1(x) \wedge C2(y) \supset p(x) \vee p(y)$.

- I.e., specialization in ILP is realized.

Related Work

- Work on disjunctive explanations in AI
  - Circumscriptive theorem prover [Helft et al. IJCAI-91]
  - Cautious explanations in causal theories [Konolige AIJ92]
  - Weakest sufficient conditions [Lin KR2000]
- View updates in disjunctive databases
  - Updates by inserting/deleting disjunctive facts in stratified DB [Grant et al. JAR93]
  - Model based view updates [Fernandez et al. JAR96]
- Knowledge assimilation
  - Abduction and ATMS/JTMS [Kakas & Mancarella ECAI90]
  - Theory change by Fagin et al. [84; 86]

In the rest of this talk, I will

- restrict our attention to the answer set semantics in extended disjunctive programs.
- analyze the property of disjunctive explanations in normal abduction.
- extend the notion of disjunctive explanations to extended abduction [Inoue & Sakama, IJCAI95; KR98].

Extended Disjunctive Program (EDP)

- Rules:

$$L_1; \ldots; L_l \leftarrow L_{l+1}, \ldots, L_m, \textbf{not}\ L_{m+1}, \ldots, \textbf{not}\ L_n$$

- Extended logic program (ELP): $k=1$.
- Normal logic program (NLP): ELP & $\forall L_i$: atom.
- Integrity constraint (IC): $l=0$.
- (Disjunctive) fact: $m=n=l\ (\geq 2)$.
- Every rule in K is either in $I(K)$ or in $F(K)$,

  where $F(K)$ denotes the facts in K and $I(K) = K - F(K)$.

Answer Sets for EDP (1)

- *Answer set semantics* [Gelfond & Lifschitz, NGC91]

I. When K is an EDP without ***not*** ($m=n$) ,

S is an **answer set** of K if

S is a minimal set satisfying the conditions:

1. For each ground rule from K :

$$L_1; \ldots; L_l \leftarrow L_{l+1}, \ldots, L_m,$$

$\{L_{l+1}, \ldots, L_m\} \subseteq \boldsymbol{S}$ implies $\{L_1, \ldots, L_l\} \cap \boldsymbol{S} \neq \phi$;

2. If ***S*** contains a pair of complementary literals, then ***S*** = ***Lit***.

Answer Sets for EDP (2)

II. When K is any EDP,

the EDP (without ***not***) K^S is obtained as follows:

A rule

$$L_1; \ldots; L_l \leftarrow L_{l+1}, \ldots, L_m$$

is in K^S iff there is a ground rule from K of the form

$$L_1; \ldots; L_l \leftarrow L_{l+1}, \ldots, L_m, \textbf{not}\ L_{m+1}, \ldots, \textbf{not}\ L_n$$

s.t. $\{L_{m+1}, \ldots, L_n\} \cap \boldsymbol{S} = \phi$.

Then, $\boldsymbol{S}$ is an **answer set** of K if

$\boldsymbol{S}$ is an answer set of K^S.

Answer Sets for GEDP (3)

- An answer set is **consistent** if it is not ***Lit***.
- An EDP is **<u>consistent</u>** if it has a consistent answer set.
- The set of all answer sets of K is denoted as $AS(K)$.
- K **<u>entails</u>** $\boldsymbol{L}$ ($K \models \boldsymbol{L}$) if $\boldsymbol{L} \in \boldsymbol{S}$ for every $\boldsymbol{S} \in AS(K)$.
- Every answer set of any EDP is minimal [Gelfond & Lifschitz]

Entailment between Programs

- R is **<u>weaker than</u>** R' ($R' \models R$)
  if for any $\boldsymbol{S} \in AS(R)$, there exists $\boldsymbol{S'} \in AS(R')$
  such that $\boldsymbol{S'} \subseteq \boldsymbol{S}$.
- For example, $\{a,b\} \models \{a\} \models \{a;b\}$.
- R and R' are **<u>equivalent</u>** if $AS(R) = AS(R')$.
- R is **<u>less presumptive (relative to *K*) than</u>** R'
  if $K \cup R' \models K \cup R$.
- R and R' are **<u>equivalent relative to *K*</u>**
  if $AS(K \cup R) = AS(K \cup R')$.

Disjunctive Explanations in Normal Abduction

- K : EDP
- A : set of **abducibles** (literals)
- $E \subseteq A$ is an **elementary explanation** of G **wrt** $\langle K,A \rangle$ if:
  1. $K \cup E \models G$
  2. $K \cup E$ is consistent.
- Any disjunction of elementary explanations is an **explanation**.

Disjunctive Hypotheses

- *K* : EDP
- *A* : set of abducibles (literals)
- *D(A)* : the set of all disjunctions formed with abducibles from *A*

- **<u>Theorem</u>:** There is a one-to-one correspondence between the **elementary explanations** of *G* **wrt** $\langle K, D(A) \rangle$ and the **disjunctive explanations** of *G* **wrt** $\langle K, A \rangle$.
- Note: The former is CNF, while the latter is DNF.

Explanation Closures

- An explanation is **minimal** if it is a weakest one.
- The **explanation closure** is the disjunction of all minimal elementary explanations.
- **Theorem:** The explanation closure wrt $\langle K, A\rangle$ is equivalent to the least presumptive elementary explanation wrt $\langle K, D(A) \rangle$.
- **Theorem:** The explanation closure wrt $\langle K, A\rangle$ preserves all and only minimal answer sets from the collection of programs with minimal elementary explanations wrt $\langle K, A\rangle$.

Explanation Closure Theorem

- **Theorem:** The explanation closure *F* wrt $\langle K, A \rangle$ preserves all and only minimal answer sets from the collection of programs with each minimal elementary explanation *E* wrt $\langle K, A \rangle$.

$$AS(K \cup \{F\}) = \mu \bigcup_{E \in ME(G)} AS(K \cup E).$$

Extended Abduction [Inoue & Sakama 95]

- Abductive logic program: $\langle K, \mathcal{A} \rangle$
  - K: EDP
  - $\mathcal{A}$: set of literals (**abducibles**)

- (P,N) is a **scenario** for $\langle K, \mathcal{A} \rangle$ if

1. P, N are sets of instances of elements from $\mathcal{A}$
2. $(K \setminus N) \cup P$ is consistent

Abductive Explanations

- A scenario *(P,N)* is an **elementary explanation** of G if: $(K \setminus N) \cup P \models G$
- A scenario *(P,N)* is an **elementary anti-explanation** of G if: $(K \setminus N) \cup P \not\models G$
- An elementary (anti-)explanation *(P,N)* is **minimal** if: for any el. (anti-)explanation *(P',N')*, $P' \subseteq P$ and $N' \subseteq N$ imply $P'=P$ and $N'=N$.

Framework of Extended Abduction

Abductive logic program
$\langle K, \Gamma \rangle$

***Positive Observation* G+**
***Negative Observation* G-**

K' = ($K \setminus N$) $\cup$ P

Abductive logic program
$\langle K', \Gamma \rangle$

K' $\models$ G+
K' $\not\models$ G-

From Extended Abduction to Normal Abduction

- $\underline{\mathcal{V}(K, \mathcal{A}) = \langle K', \mathcal{A}' \rangle}$

1. $K' = (K \setminus \mathcal{A}) \cup \{ a \leftarrow \textbf{\textit{not}}\ a' \mid a \in K \cap \mathcal{A} \}$.
2. $\mathcal{A}' = \mathcal{A} \cup \{ a' \mid a \in K \cap \mathcal{A} \}$.

- *a'* represents the *deletion* of *a*.

Theorem [Inoue 2000]

- *(P,N)* is a minimal elementary explanation of *G* wrt $<K, \mathcal{A}>$ under extended abduction

  if and only if

  E is a minimal elementary explanation of *G* wrt $\nu(K, \mathcal{A})$ under normal abduction, where

  $P = \{ a \mid a \in E \cap \mathcal{A} \}$,

  $N = \{ a \mid a' \in E \}$.

Translation of Anti-Explanations

- (P,N) is a minimal elementary anti-explanation of G wrt $\langle K, \mathcal{A} \rangle$ <u>*if and only if*</u>

  (E, ϕ) is a minimal elementary anti-explanation of G wrt $\nu\ (K, \mathcal{A})$, where

  $$P = \{\, a \mid a \in E \cap \mathcal{A} \,\},$$

  $$N = \{\, a \mid a' \in E \,\}.$$

- Anti-explanations can be converted into explanations by associating G with the new rule:

$$G' \leftarrow \textbf{\textit{not}}\ G.$$

Extended Disjunctive Abduction

- (P,N) is a **d-scenario** for $\langle K, \mathcal{A} \rangle$ if

  1. P is a set of instances of elements from $\mathcal{A}$,
  2. N is a set of instances of elements from $D(\mathcal{A})$,
  3. $(K \setminus N) \cup P$ is consistent.

- A d-scenario (P,N) is an elementary **d-explanation** of G if: $(K \setminus N) \cup P \models G$.
- A d-scenario (P,N) is an elementary **d-anti-explanation** of G if: $(K \setminus N) \cup P \not\models G$.

⌘ An el. d-(anti-)explanation (P,N) is **minimal** if:
for any el. d-(anti-)explanation (P',N'),
$P' \models P$ and $N' \models N$ imply $P' \models P$ and $N' \models N$.

From Extended Disjunctive Abduction to Normal Abduction

- $\underline{\mathcal{V}^{\mathbf{d}}(K, \mathcal{A}) = \langle K', \mathcal{A}' \rangle}$

1. $K' = (K \setminus D(\mathcal{A}))$

   $\cup \{ a \leftarrow \mathbf{not}\ del(a) \mid a \in K \cap D(\mathcal{A}) \}.$

2. $\mathcal{A}' = \mathcal{A} \cup \{ del(a) \mid a \in K \cap D(\mathcal{A}) \}.$

Explanation Closure Theorem

- **<u>Theorem</u>:** The explanation closure *F* wrt $\langle K, A\rangle$ preserves all and only minimal answer sets from the collection of programs with each minimal elementary d-explanation *(P,N)* wrt $\langle K, A\rangle$.

$$AS^{-del}(\nu^{d}(K) \cup \{F\}) = \mu \bigcup_{(P,N)\in ME^{d}(G)} AS((K \text{ ¥ } N) \cup P).$$

Anti-Explanation Closure Theorem

- **Theorem:** The anti-explanation closure *H* wrt ⟨*K, A*⟩ preserves all and only minimal answer sets from the collection of programs with each minimal el. d-anti-explanation *(P,N)* wrt ⟨*K, A*⟩.

$$AS^{-del}(\nu^{d}(K) \cup \{H\}) = \mu \bigcup_{(P,N)\in MEA^{d}(G)} AS((K \text{ ¥ } N) \cup P).$$

Indefinite Removal Example

- **Insert** p in view update from

$$p \leftarrow \textbf{not}\ a,$$
$$p \leftarrow \textbf{not}\ b,$$
$$a\ ,$$
$$b\ ,$$

with $\{a, b\}$: (removable) abducibles.

- Solutions (minimal el. explanation):

$(P1,N1)=(\{\},\{a\}),\ (P2,N2)=(\{\},\{b\}).$

Indefinite Removal Example

$$p \leftarrow \textbf{\textit{not}}\ a,$$
$$p \leftarrow \textbf{\textit{not}}\ b,$$
$$a \leftarrow \textbf{\textit{not}}\ del(a),$$
$$b \leftarrow \textbf{\textit{not}}\ del(b).$$

- Solutions (explanation closure):

$$F = del(a)\ ;\ del(b).$$

Conclusion

- Abduction from $\langle K, D(A) \rangle$ is worth considering, as it can be applied to knowledge assimilation, view updates, skeptical reasoning, specialization, etc.
- Abduction from $\langle K, D(A) \rangle$ is easy to implement by considering disjunctive explanations which are constructed with elementary explanations in abduction from $\langle K, A \rangle$.
- Extended abduction from $\langle K, D(A) \rangle$ is similarly realized by converting it to normal abduction. Updates in disjunctive programs can also be formalized in this way.