COUNTERFACTUAL REASONING IN ARGUMENTATION FRAMEWORKS

CHIAKI SAKAMA

WAKAYAMA UNIVERSITY, JAPAN

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BACKGROUND & GOAL

- **A counterfactual** is a conditional statement representing what would be the case if its premise were true (but it is not true in fact).
  "*If COMMA 2014 were not held in Scotland, you would not be here now*."
- A formal model of counterfactuals has been studied by Stalnaker (1968) and Lewis (1973) based on the **possible world semantics**.
- Counterfactuals are popularly used in dialogue, argument or dispute, while little attention has been paid on it in formal argumentation.
- The purpose of this study is to provide an argumentation-theoretic interpretation of counterfactuals.

ARGUMENTATION FRAMEWORK

(DUNG 1995; CAMINADA & GABBAY 2009)

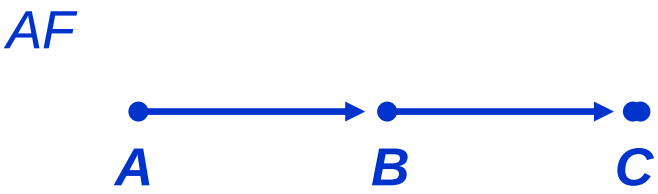
- Let ***U*** be the **universe** of all possible arguments.
  An **argumentation framework (AF)** is a pair (Ar, att) where Ar is a finite subset of ***U*** and $att \subseteq Ar \times Ar$.
  An argument A **attacks** an argument B iff $(A,B) \in att$.

- A **indirectly attacks** (resp. **indirectly defends**) B if there is an odd-length (resp. even-length, non-zero) path from A to B in a directed graph associated with AF.

- A **labelling** of AF is a function $\mathcal{L}: Ar \rightarrow \{ in, out, undec \}$.

- **Complete, (semi-)stable, grounded,** and **preferred labelling** are defined as usual. We simply say “labelling” to indicate one of these 5 labellings.

COUNTERFACTUAL REASONING IN ARGUMENTATION FRAMEWORK



- "If *A* were rejected, then *B* would be accepted."
  "If *A* were rejected, then *C* would be rejected."
  "If *B* were accepted, then *C* would be rejected."
- Modify *AF* to *AF'* in a way that
  an argument *A* that is accepted in *AF* is rejected in *AF'* ;
  or an argument *B* that is rejected in *AF* is accepted in *AF'*.

MODIFICATION OF AF

- Let $AF=(Ar, att)$ and $A \in Ar$. Define

  $AF^{c}_{+A} = (Ar, att \setminus \{ (X, A) \mid X \in Ar \})$
  (removing every attack relation attacking A)

  $AF^{c}_{-A} = (Ar \cup \{X\}, att \cup \{ (X,A) \})$ where $X \in U \setminus Ar$ and $U \setminus Ar \neq \phi$
  (introducing an argument that attacks A)

- $\mathcal{L}(A)=in$ for every labelling $\mathcal{L}$, if any, in AF^{c}_{+A}

- $\mathcal{L}(A)=out$ for every labelling $\mathcal{L}$, if any, in AF^{c}_{-A}

- AF^{c}_{+A} and AF^{c}_{-A} are simply written AF^{c}
  if an argument A is clear in the context.

COUNTERFACTUALS(CF) IN AF

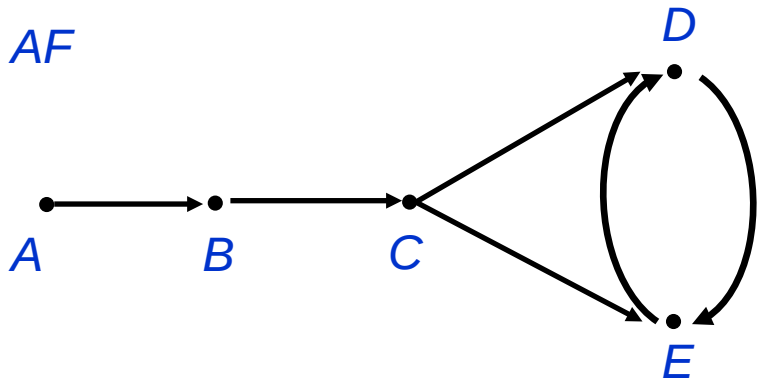
Let $AF=(Ar, att)$ and $A, B \in Ar$, and $\ell \in \{ in, out \}$. Define

- $in(A) \Box\!\!\rightarrow \ell(B)$ is true in AF if $\mathcal{L}(B)=\ell$ in every labelling $\mathcal{L}$ of AF^{c}_{+A}
- $in(A) \Diamond\!\!\rightarrow \ell(B)$ is true in AF if $\mathcal{L}(B)=\ell$ in some labelling $\mathcal{L}$ of AF^{c}_{+A}
- $out(A) \Box\!\!\rightarrow \ell(B)$ is true in AF if $\mathcal{L}(B)=\ell$ in every labelling $\mathcal{L}$ of AF^{c}_{-A}
- $out(A) \Diamond\!\!\rightarrow \ell(B)$ is true in AF if $\mathcal{L}(B)=\ell$ in some labelling $\mathcal{L}$ of AF^{c}_{-A}

where “labelling” means one of the 5 labellings of *AF*.

- $in(A) \Box\!\!\rightarrow in(B)$ is read “if *A* were accepted then *B* would be accepted.”
- $in(A) \Box\!\!\rightarrow out(B)$ is read “if *A* were accepted then *B* would be rejected.”
- $in(A) \Diamond\!\!\rightarrow in(B)$ is read “if *A* were accepted then *B* might be accepted.”
- $in(A) \Diamond\!\!\rightarrow out(B)$ is read “if *A* were accepted then *B* might be rejected.”

EXAMPLE



- *AF* has the complete labelling: { *in*(*A*), *out*(*B*), *in*(*C*), *out*(*D*), *out*(*E*) }.
- The following CFs hold in *AF*:

out(*A*) $\Box\!\!\rightarrow$ in(*B*) in(*B*) $\Box\!\!\rightarrow$ out(*C*)

out(*C*) $\Diamond\!\!\rightarrow$ in(*D*) out(*C*) $\Diamond\!\!\rightarrow$ out(*E*)

out(*C*) $\Diamond\!\!\rightarrow$ out(*D*) out(*C*) $\Diamond\!\!\rightarrow$ in(*E*)

FORMAL PROPERTIES (1)

Let $AF=(Ar, att)$ and $A, B \in Ar$, and $\ell \in \{ in, out \}$.

- $\ell_1 (A) \; \Box\!\!\rightarrow \ell_2(B)$ implies $\ell_1 (A) \; \Diamond\!\!\rightarrow \ell_2(B)$
- CFs are reflexive:
  $\ell (A) \; \Box\!\!\rightarrow \ell(A)$ is true in AF.
  $\ell(A) \; \Diamond\!\!\rightarrow \ell(A)$ is true in AF whenever AF^{c} has a labelling.
- CFs with true antecedent:

  - If $\mathcal{L}(A)= \ell_1$ and $\mathcal{L} (B)= \ell_2$ in every complete labelling $\mathcal{L}$ of AF, then $\ell_1 (A) \; \Box\!\!\rightarrow \ell_2(B)$ is true in AF.

  - If $\mathcal{L}(A)= \ell_1$ in every complete labelling $\mathcal{L}$ of AF and $\mathcal{L} (B)= \ell_2$ in some complete labelling $\mathcal{L}$ of AF, then $\ell_1 (A) \; \Diamond\!\!\rightarrow \ell_2(B)$ is true in AF.

FORMAL PROPERTIES (2)

- Modus Ponens:
  - If $\mathcal{L}(A)= \ell_1$ in every complete labelling $\mathcal{L}$ of *AF*
    and $\ell_1 (A) \Box\!\!\rightarrow \ell_2(B)$ is true in *AF,*
    then $\mathcal{L}'(B)= \ell_2$ in every complete labelling $\mathcal{L}'$ of *AF*.

  - If $\mathcal{L}(A)= \ell_1$ in every complete labelling $\mathcal{L}$ of *AF*
    and $\ell_1 (A) \Diamond\!\!\rightarrow \ell_2(B)$ is true in *AF,*
    then $\mathcal{L}'(B)= \ell_2$ in some complete labelling $\mathcal{L}'$ of *AF*.

- Modus Tollens:
  If $\ell_1 (A) \Box\!\!\rightarrow \ell_2(B)$ is true in *AF* and $\mathcal{L}(B) \neq \ell_2$ for any complete labelling $\mathcal{L}$ of *AF*, then $\mathcal{L}'(A) \neq \ell_1$ for any complete labelling $\mathcal{L}'$ of *AF.*

- Others

  - If $\ell_1 (A) \Box\!\!\rightarrow \ell_2(B)$ is true in *AF,* then $\ell_1 (A) \not\Diamond\!\!\rightarrow \overline{\ell_2}(B)$ is true in *AF.*

  - If $\ell_1 (A) \Diamond\!\!\rightarrow \ell_2(B)$ is true in *AF,* then $\ell_1 (A) \not\Box\!\!\rightarrow \overline{\ell_2}(B)$ is true in *AF.*

  where $\overline{\ell}$ is "out" (resp. "in") if ℓ is "in" (resp. "out").

COUNTERFACTUAL FALLACIES

- Fallacy of strengthening the antecedent:

  $\ell_1(A) \boxrightarrow \ell_2(B)$ in *AF* does not imply
  $\ell_1(A) \wedge \ell_3(C) \boxrightarrow \ell_2(B)$ in *AF* in general.

- Fallacy of transitivity:

  $\ell_1(A) \boxrightarrow \ell_2(B)$ and $\ell_2(B) \boxrightarrow \ell_3(C)$ in *AF* do not imply
  $\ell_1(A) \boxrightarrow \ell_3(C)$ in *AF* in general.

- Fallacy of contraposition:

  $\ell_1(A) \boxrightarrow \ell_2(B)$ in *AF* does not imply
  $\overline{\ell_2}(B) \boxrightarrow \overline{\ell_1}(A)$ in *AF* in general.

The above results also hold by replacing $\boxrightarrow$ with $\diamond\!\!\rightarrow$.

COUNTERFACTUAL DEPENDENCIES

- "An event φ depends causally on another event ψ iff both $\varphi \mathrel{\Box\!\!\rightarrow} \psi$ and $\neg\varphi \mathrel{\Box\!\!\rightarrow} \neg\psi$ hold" (Lewis 1973).

- Counterfactual dependencies

  $\ell_1(A) \overset{c}{\Box\!\!\rightarrow} \ell_2(B)$ is true in *AF*
  if both $\ell_1(A) \mathrel{\Box\!\!\rightarrow} \ell_2(B)$ and $\overline{\ell_1}(A) \mathrel{\Box\!\!\rightarrow} \overline{\ell_2}(B)$ hold in *AF*
  where $\ell_1, \ell_2 \in \{ in, out \}$.

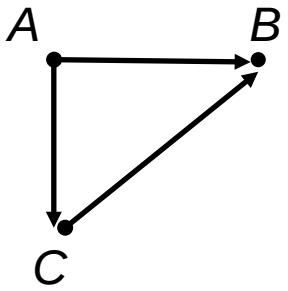
- Formal properties

  - $\ell_1(A) \overset{c}{\Box\!\!\rightarrow} \ell_2(B)$ implies $\ell_1(A) \mathrel{\Box\!\!\rightarrow} \ell_2(B)$, but not vice versa.

  - If $\ell(A) \overset{c}{\Box\!\!\rightarrow} \ell(B)$ is true in *AF*, then *A* indirectly defends *B*.

  - If $\ell(A) \overset{c}{\Box\!\!\rightarrow} \overline{\ell}(B)$ is true in *AF*, then *A* indirectly attacks *B*.

PREEMPTION



- in(A) $\square\!\!\rightarrow$ out(B) but out(A) $\square\!\!\not\rightarrow$ in(B) imply in(A) $\square \overset{c}{\not\rightarrow}$ out(B)
- in(C) $\square\!\!\rightarrow$ out(B) but out(C) $\square\!\!\not\rightarrow$ in(B) imply in(C) $\square \overset{c}{\not\rightarrow}$ out(B)
- Thus out(B) causally depends on neither in(A) nor in(C).
- The existence of *A* is the actual cause of rejecting *B*. On the other hand, if *A* were rejected then *C* would be accepted, which results in rejecting *B*. Thus, *C* is a “potential” alternative cause of rejecting *B*.
- in(C) $\square\!\!\rightarrow$ out(B) represents that in(C) is a potential cause of out(B) but is **preempted** by the actual cause in(A).

MODAL INTERPRETATION

- $\Box\ell(A) \stackrel{\text{def}}{=} \overline{\ell}(A) \,\Box\!\!\rightarrow \text{in}(\bot)$ and $\Diamond\ell(A) \stackrel{\text{def}}{=} \ell(A) \,\Box\!\!\not\rightarrow \text{in}(\bot)$
- For instance, $\Box\text{in}(A)$ ("A is necessarily accepted")
  iff $\text{out}(A) \,\Box\!\!\rightarrow \text{in}(\bot)$
  ("inconsistency would be accepted if A were rejected")
- Example

  A="$\sqrt{2}$ is an irrational number"

  The validity of the argument A is proven by showing inconsistency under the assumption that $\sqrt{2}$ were a rational number.

  $\Rightarrow \Box\text{in}(A)$ is proved by showing $\text{out}(A) \,\Box\!\!\rightarrow \text{in}(\bot)$

COMPLEXITY

- Given an *AF*, the problem of deciding whether $in(A) \Box\!\!\rightarrow \ell(B)$ (resp. $in(A) \Diamond\!\!\rightarrow \ell(B)$) holds or not in *AF* is equivalent to deciding whether $\mathcal{L}(B) = \ell$ in every (resp. some) labelling $\mathcal{L}$ of AF^{c}_{+A}
- Likewise, deciding whether $out(A) \Box\!\!\rightarrow \ell(B)$ (resp. $out(A) \Diamond\!\!\rightarrow \ell(B)$) holds or not in *AF* is equivalent to deciding whether $\mathcal{L}(B) = \ell$ in every (resp. some) labelling $\mathcal{L}$ of AF^{c}_{-A}
- Therefore, complexities of CF under the operator $\Box\!\!\rightarrow$ (resp. $\Diamond\!\!\rightarrow$) are equivalent to those of skeptical (resp. credulous) reasoning of an argument under argumentation semantics.

POTENTIAL APPLICATIONS

- CFs are popularly used in dialogue or dispute, so the framework would be useful for realizing CF reasoning in **dialogue systems based on AF**.
- CFs are used in diagnosis in which assumptions are introduced for explaining the observed misbehavior of a device.
  Thus, CFs could be used for **analytic tools in AF**.
- When one wants to have a desired outcome which would not be achieved by true arguments, one would reason counterfactually.
  In this case, CFs would be used for **building false arguments in debate/discussion games**.
- CF studied here is based on abstract AF and it will be applied to **instantiated AFs** based on particular representation languages.