Specifying Transactions for Extended Abduction

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Abstract

Extended abduction introduced by Inoue and Sakama (1995) generalizes traditional abduction in the sense that it can compute *negative explanations* by removing hypotheses from a nonmonotonic background theory, rather than only adding them. Also, it has a mechanism of computing *anti-explanations* to *un*explain negative observations. Such extended abduction not only enhances reasoning ability of traditional abduction but has useful applications to nonmonotonic theory change. In this paper, we study the computational aspect of extended abduction. Given a background theory written in nonmonotonic logic programming, we introduce its *transaction program* for computing extended abduction. A transaction program is a set of nondeterministic production rules that specify addition and deletion of abductive hypotheses. Abductive explanations are computed by the fixpoint of a transaction program using a bottom-up model generation procedure. In the context of databases, a transaction program provides a declarative specification of database update.

1 INTRODUCTION

Given a background knowledge base K and an observation G, abduction computes an explanation E of G satisfying

$$K \cup E \models G$$

where $K \cup E$ is consistent. This is the traditional logical framework of abduction widely accepted in AI (Poole, 1988). However, the above framework of abduction is not sufficient when the background knowledge base is *nonmonotonic*. For example, consider the knowledge base written in nonmonotonic logic programming:

$$\begin{aligned} K: \quad & flies(x) \leftarrow bird(x), not\ ab(x), \\ & ab(x) \leftarrow broken\text{-}wing(x), \\ & bird(tweety) \leftarrow , \\ & bird(opus) \leftarrow , \\ & broken\text{-}wing(tweety) \leftarrow . \end{aligned}$$

If we observe that *tweety* flies, there is a good reason to assume that the wound has already healed. Then, the fact $broken\text{-}wing(tweety)$ must be *removed* from the knowledge base to explain the observation $flies(tweety)$.

The traditional abduction cannot characterize such a situation. That is, abduction introduces hypotheses to a knowledge base, but once they are included, any hypothesis cannot be removed from the knowledge base. To cope with this problem, Inoue and Sakama (1995) introduced the notion of "negative explanations". Given a background knowledge base K and an observation G, a set F of formulas is called a *negative explanation* of G if

$$K \setminus F \models G$$

where $K \setminus F$ is consistent. An explanation E satisfying $K \cup E \models G$ is then called a *positive explanation*.

On the other hand, suppose that we later notice that *opus* does not fly any more. Since $flies(opus)$ is entailed by K, we now have to revise the knowledge base to block the derivation of $flies(opus)$ by assuming $broken\text{-}wing(opus)$, for instance. This situation is characterized by the notion of "anti-explanations" which are used to *un*explain observations (Inoue and Sakama, 1995). Given a background knowledge base

K and an observation G, a set E of formulas is called a *(positive) anti-explanation* of G if

$$K \cup E \not\models G,$$

and a set F of formulas is called a *negative anti-explanation* of G if

$$K \setminus F \not\models G.$$

These extensions of traditional abduction are useful when we consider nonmonotonic theories as background knowledge bases. Firstly, the introduction of negative explanations is natural, since it is often the case that deletion of formulas introduces new formulas in nonmonotonic reasoning. In fact, positive and negative explanations play a complementary role in accounting for an observation in nonmonotonic theories. Secondly, the introduction of anti-explanations is necessary when one wants to account for *negative observations*. In this respect, traditional abduction is concerned with explaining *positive observations* only. Negative observations are often perceived in real-life situations. For instance, many *inductive concept learning* systems consider both positive and negative examples to construct a hypothetical theory. In this sense, the notion of anti-explanations plays a dual role to explanations. Thirdly, extended abduction not only enhances reasoning ability of traditional abduction, but is essential to theories of *nonmonotonic theory change* and *abductive theory revision* (Inoue and Sakama, 1995). Useful applications of extended abduction thus include *view update* in deductive databases, *contradiction removal* as well as *diagnosis* and *repair*. For example, model-based diagnosis (Reiter, 1987) constructs a diagnostic hypothesis by identifying a minimal set of fault components Δ from the set of system components $COMP$ such that (i)

$$H = \{Ab(c) \mid c \in \Delta\} \cup \{\neg Ab(c) \mid c \in COMP \setminus \Delta\}$$

and (ii) H is consistent with the system description and observations. Suppose that, after repairing some components Δ' in Δ, we get new observations. Then,

$$H' = (H \cup E) \setminus F$$

with $E = \{\neg Ab(c) \mid c \in \Delta'\}$ and $F = \{Ab(c) \mid c \in \Delta'\}$ becomes a new diagnosis. Here, positive and negative explanations are necessary at the same time. Recently, Buccafurri *et al.* (1997) apply extended abduction to formalize a system repair problem in the context of model checking, which is a successful technique for the verification of concurrent programs and protocols.

In this paper, we study the computational aspect of extended abduction. We consider knowledge bases written in normal logic programs, and introduce a program transformation which produces a *transaction program*. A transaction program is a kind of disjunctive logic program that specifies addition and deletion of abductive hypotheses. Abductive explanations are computed by the fixpoint of the transaction program using bottom-up model generation procedures like (Bry, 1990; Inoue and Sakama, 1996; Aravindan and Baumgartner, 1997). The correctness of the proposed method is shown for the class of acyclic programs. In the context of databases, a transaction program provides a declarative specification of database update. In a special case, a transaction program gives a *revision program* in the sense of Marek and Truszczynski (1994).

The rest of this paper is organized as follows. In Section 2, we outline our extended abductive framework. In Section 3, we introduce a transformation of abductive logic programs into transaction programs, and present a fixpoint computation for extended abduction. Section 4 addresses comparisons with related work, and Section 5 concludes the paper.

2 ABDUCTIVE FRAMEWORK

An abductive framework considered in this paper is defined as follows.

A *normal logic program* (NLP) is a finite set of rules of the form:

$$A_0 \leftarrow A_1, \ldots, A_m, \mathit{not}\, A_{m+1}, \ldots, \mathit{not}\, A_n \quad (1)$$

($n \geq m \geq 0$), where each A_i is an atom and *not* denotes *negation as failure*. The left-hand side of a rule is the *head*, and the right-hand side is the *body*. We allow a rule with an empty head, which is called an *integrity constraint*. In this paper, the empty head of any integrity constraint is represented by the special atom $\bot$. A rule with an empty body ($n = 0$) is called a *fact*. Each fact $A \leftarrow$ is identified with the atom A.

A program (rule, atom, term) is *ground* if it contains no variable. An NLP P is *acyclic* if there is a *level mapping* from the ground atoms of P to the natural numbers, such that for any ground rule (1) from P, the level of A_0 is higher than the level of every A_i ($i = 1, \ldots, n$) (Apt and Bezem, 1991). P is *Horn* if no rule in P contains negation as failure, that is, $m = n$ holds for every rule of the form (1) in P. P is *definite* if it is a Horn program without integrity constraints.

The semantics of NLPs is given by the *stable model semantics* (Gelfond and Lifschitz, 1988). Given any NLP P and a set M of ground atoms, the ground

Horn NLP P^M is defined as follows: a rule

$$A_0 \leftarrow A_1, \ldots, A_m$$

is in P^M iff there is a ground rule of the form (1) from P such that $\{A_{m+1}, \ldots, A_n\} \cap M = \emptyset$. Note that P^M is a possibly infinite set of *not*-free ground rules. Then, P^M has the unique minimal model, i.e., the least model. If the least model of P^M is identical to M, M is called a *stable model* of P. A stable model is *consistent* if it does not contain $\bot$, that is, it satisfies every integrity constraint. An NLP is *consistent* if it has a consistent stable model. For a consistent NLP P and an atom L, P *entails* L (written $P \models L$) if L is included in every stable model of P; otherwise $P \not\models L$. Note that any acyclic program has a unique stable model (Gelfond and Lifschitz, 1988).

An *abductive logic program* (ALP) is defined as a pair $\langle P, \mathcal{A} \rangle$, where P is an NLP and $\mathcal{A}$ is a set of atoms called *abducibles*. Any instance A of an element from $\mathcal{A}$ is also called an *abducible* and is written as $A \in \mathcal{A}$. Without loss of generality, we assume that any rule (1) having an abducible in its head ($A_0 \in \mathcal{A}$) is always a fact.[1] An ALP $\langle P, \mathcal{A} \rangle$ is *acyclic* (resp. *Horn*, *ground*) if P is acyclic (resp. Horn, ground).

Definition 2.1 Let $\langle P, \mathcal{A} \rangle$ be an ALP, and G a ground atom. A pair (E, F) is an *explanation* (resp. *anti-explanation*) of an *observation* G wrt $\langle P, \mathcal{A} \rangle$ if

1. $(P \cup E) \setminus F \models G$ (resp. $(P \cup E) \setminus F \not\models G$),
2. $(P \cup E) \setminus F$ is consistent, and
3. $E \subseteq \mathcal{A}$ and $F \subseteq \mathcal{A}$.

For an (anti-)explanation (E, F), E is also called a *positive (anti-)explanation* if $F = \emptyset$; F is called a *negative (anti-)explanation* if $E = \emptyset$; and (E, F) is called a *mixed (anti-)explanation* if $E \neq \emptyset$ and $F \neq \emptyset$.

An (anti-)explanation (E, F) of G is *minimal* if for any (anti-)explanation (E', F') of G, $E' \subseteq E$ and $F' \subseteq F$ imply $E' = E$ and $F' = F$.

In this paper, minimal explanations are of particular interest, and (anti-)explanations mean minimal (anti-)explanations unless stated otherwise. Note that $E \cap F = \emptyset$ holds for any minimal (anti-)explanation (E, F).

Example 2.1 Consider the introductory program K in Section 1. Let $\langle K, \mathcal{A} \rangle$ be an ALP where $\mathcal{A} = \{ \mathit{broken\text{-}wing}(x) \}$. Then, $G_1 = \mathit{flies}(\mathit{tweety})$ has the explanation $(E, F) = (\emptyset, \{\mathit{broken\text{-}wing}(\mathit{tweety})\})$, while $G_2 = \mathit{flies}(\mathit{opus})$ has the anti-explanation $(\{\mathit{broken\text{-}wing}(\mathit{opus})\}, \emptyset)$.

[1] If there is a rule $A \leftarrow \Gamma$ with an abducible A and a non-empty body Γ, then A is made a non-abducible by introducing a rule $A \leftarrow A'$ with a new abducible A'.

3 COMPUTING EXTENDED ABDUCTION

In this section, we provide a method of computing extended abduction. In Sections 3.1 and 3.2, programs are assumed to be ground, and results are extended to non-ground programs in Section 3.3.

3.1 TRANSACTION PROGRAM

We first introduce the notion of *transaction programs*, which are obtained by transforming abductive logic programs. This transformation is motivated to get rules for what must be in the knowledge base and what must be outside it for any observation to hold.

For any atom A, formulas $\mathit{in}(A)$ and $\mathit{out}(A)$ are called *T-literals* (or *literal* for short). A *transaction program* consists of *productions* of the form:

$$L \rightarrow \alpha_1 \mid \ldots \mid \alpha_k, \tag{2}$$

where L is a literal and α_i is a sequence of literals.[2] Here, the vertical line "|" is read "or" meaning a disjunction, while a sequence means a conjunction of literals. In a production of the form (2), the left-hand side L is called the *antecedent* and the right-hand side $\alpha_1 \mid \ldots \mid \alpha_k$ is called the *consequent* of the production.

Definition 3.1 Let $\langle P, \mathcal{A} \rangle$ be a ground ALP. Then, the *transaction program* τP *of* P *(relative to* $\mathcal{A}$*)* is defined as follows.

1. Let H be a non-abducible atom ($H \notin \mathcal{A}$) and

$$\begin{aligned} H &\leftarrow B_1, \\ &\vdots \\ H &\leftarrow B_k, \end{aligned}$$

be all rules in P with H in the head, where

$$B_i = A_1, \ldots, A_{m_i}, \mathit{not}\, C_1, \ldots, \mathit{not}\, C_{n_i}$$

($1 \leq i \leq k,\ m_i \geq 0,\ n_i \geq 0$). Then, these rules are transformed to the following productions:

$$R_{in}: \qquad \mathit{in}(H) \rightarrow \mathit{in}(B_1) \mid \ldots \mid \mathit{in}(B_k), \tag{3}$$

[2] In non-ground transaction programs, productions may contain equalities or inequalities in their antecedents or consequents (see Definition 3.4).

$$R_{out}: \quad \begin{array}{l} out(H) \rightarrow out(B_1), \\ \quad \vdots \\ out(H) \rightarrow out(B_k), \end{array} \tag{4}$$

where for $i = 1, \ldots, k$, $in(B_i)$ and $out(B_i)$ are defined as follows. If B_i is empty, $in(B_i) = \varepsilon$; otherwise $in(B_i)$ is:

$$in(A_1), \ldots, in(A_{m_i}), out(C_1), \ldots, out(C_{n_i}).$$

If B_i is empty, $out(B_i) = false$; else $out(B_i)$ is:

$$out(A_1) \mid \ldots \mid out(A_{m_i}) \mid in(C_1) \mid \ldots \mid in(C_{n_i}).$$

When $H = \bot$ (integrity constrains), only R_{out} is included in τP.

2. When $H \leftarrow$ is in P for $H \in \mathcal{A}$ (note in this case that there is no rule with H in the head and with a non-empty body by the assumption [footnote 1]), it is transformed to

$$R_{in}: \quad in(H) \rightarrow \varepsilon,$$

and we define that R_{out} is empty.

3. Let A be any atom that does not appear in the head of any rule in P. For every such atom A, introduce the production:

$$out(A) \rightarrow \varepsilon \tag{5}$$

to τP. Moreover, when A is a non-abducible ($A \notin \mathcal{A}$), τP also includes:

$$in(A) \rightarrow false. \tag{6}$$

Each production generated in the first step specifies abductive specifications such that to explain H, one of $B_1, \ldots, B_k$ must be explained (3), while to unexplain H, every $B_1, \ldots, B_k$ must be unexplained (4). The meanings of $in(B_i)$ and $out(B_i)$ are obvious, that is, introduction and removal of B_i respectively, which correspond to explaining/unexplaining B_i. When B_i is empty, there is nothing to be introduced and $in(B_i) = \varepsilon$. Here, the empty string ε is logically equivalent to $true$ and $\{\varepsilon\} \cup S$ is identical to S for any set S. On the other hand, $out(B_i) = false$ means that removing a non-abducible H with the empty body is impossible. Note that, for integrity constraints, the removal of $\bot$ means satisfaction of all constraints. In the second step, when an abducible H is in P, nothing is required to add H, while no further transaction is requested to remove H. In the third step, additional productions (5) mean that the removal of A implies no requirement if P has no rule with A in the head, while productions (6) mean that the addition of A is impossible.

Example 3.1 Let $\langle P, \mathcal{A} \rangle$ be an ALP, where

$$\begin{array}{ll} P: & p \leftarrow q, not\, a, \\ & p \leftarrow b, not\, r, \\ & q \leftarrow not\, c, \\ & r \leftarrow d, \\ & c \leftarrow, \\ & d \leftarrow, \\ \mathcal{A}: & a,\ b,\ c,\ d. \end{array}$$

Then, τP becomes

$$\begin{array}{l} in(p) \rightarrow in(q), out(a) \mid in(b), out(r), \\ out(p) \rightarrow out(q) \mid in(a), \\ out(p) \rightarrow out(b) \mid in(r), \\ in(q) \rightarrow out(c), \\ out(q) \rightarrow in(c), \\ in(r) \rightarrow in(d), \\ out(r) \rightarrow out(d), \\ in(c) \rightarrow \varepsilon, \\ in(d) \rightarrow \varepsilon, \\ out(a) \rightarrow \varepsilon, \\ out(b) \rightarrow \varepsilon. \end{array}$$

3.2 FIXPOINT COMPUTATION

For computing extended abduction, we consider fixpoint computation of transaction programs. The $\mathbf{T}_{\tau P}$ *fixpoint operator* which we define next is similar to that given by Inoue and Sakama (1996) for disjunctive logic programs. The only difference between the operator below and that in (Inoue and Sakama, 1996) lies in the notion of *marking*. Since each production can also be viewed as a *rewriting rule*, once a literal L is rewritten it is marked as $L*$. In this way, it is recognized that the requirement of addition/deletion of L has been translated into requests of another atoms' addition/deletion.

Formally, for T-literals $in(A)$ and $out(A)$, formulas $in(A)*$ and $out(A)*$ are also called *T-literals* (or *literals* for short). T-literals with the "$*$" symbol are called *marked literals*, while other literals are *unmarked*. Given a transaction program τP, let $\mathcal{B}$ be the set of ground T-literals in the language of τP, i.e., the Herbrand base. A *transaction* is a subset of $\mathcal{B}$, i.e.,

an Herbrand interpretation. Each transaction specifies which atoms should be added or deleted. A transaction S is *coherent* if for any atom A, S does not include any pair of $\{in(A), out(A)\}$, $\{in(A)*, out(A)\}$, $\{in(A), out(A)*\}$ and $\{in(A)*, out(A)*\}$.

A transaction S *satisfies* a T-literal L (written $S \models L$), where L is either $in(A)$ or $out(A)$, if either $L \in S$ or $L* \in S$ holds. For a conjunction of literals, we also write $S \models L_1, \ldots, L_n$ if $S \models L_i$ for every $i = 1, \ldots, n$. A transaction S *satisfies* a production $(L \rightarrow \alpha_1 \mid \ldots \mid \alpha_k)$ if $S \models L$ implies $S \models \alpha_i$ for some i $(1 \leq i \leq k)$.

Now, we define a mapping over sets of transactions. For this purpose, two operations *marking* $(*)$ and *filtering* $(\overset{*}{-})$ are defined as follows. For transactions I and J,

$$I * J = (I \setminus J) \cup \{L* \mid L \in I \cap J\},$$
$$I \overset{*}{-} J = \{L \in I \mid L \notin J \text{ and } L* \notin J\}.$$

Intuitively, $I*J$ denotes that each literal in J is marked in I, and $I \overset{*}{-} J$ means that a literal in I is removed from I if it is included in J in either marked or unmarked form. Also, for a conjunction of literals $F = L_1, \ldots, L_m$, we denote the set of its conjuncts as $\{F\} = \{L_1, \ldots, L_m\}$.

Definition 3.2 Let τP be a ground transaction program, $\mathbf{I}$ a set of transactions. Then the mapping $\mathbf{T}_{\tau P} : 2^{2^{\mathcal{B}}} \rightarrow 2^{2^{\mathcal{B}}}$ is defined as

$$\mathbf{T}_{\tau P}(\mathbf{I}) = \bigcup_{I \in \mathbf{I}} \{\, J \in T_{\tau P}(I) \mid J \text{ is coherent} \,\}$$

where the mapping $T_{\tau P} : 2^{\mathcal{B}} \rightarrow 2^{2^{\mathcal{B}}}$ is defined as follows. For any transaction I, $T_{\tau P}(I)$ is equal to:

$$\begin{cases} \emptyset\,, & \text{if } I \models L \text{ for some production} \\ & \qquad (L \rightarrow false) \text{ in } P\,; \\ \\ \{\ (I * J) \cup (K \overset{*}{-} I) \ \mid & \\ \quad J = \{L_i \mid R_i \in \mathbf{R}\} \text{ and } K = \bigcup_{R_i \in \mathbf{R}} \{\alpha^i\}, & \\ \quad \text{where } \mathbf{R} \text{ is the set of productions} & \\ \quad \text{of the form:} & \\ \qquad R_i : \ (L_i \rightarrow F_1 \mid \ldots \mid F_{k_i}) & \\ \quad \text{in } P \text{ such that } I \models L_i \text{ and that} & \\ \qquad \{F_j\} \overset{*}{-} I \neq \emptyset \text{ for every } j = 1, \ldots, k_i, & \\ \quad \text{and } \alpha^i \text{ is one of } F_1, \ldots, F_{k_i} \,\}\,, \text{ otherwise}. & \end{cases}$$

In particular, $\mathbf{T}_{\tau P}(\emptyset) = \emptyset$.

The intuitive reading of Definition 3.2 is as follows. If a transaction I does not satisfy a production with $false$ in the consequent, then I is pruned. Else, if there is a production R_i that is not satisfied by I (i.e., I satisfies the antecedent of R_i but does not satisfy the consequent of R_i), then I is expanded by adding each single disjunct F_j in the consequent for every such R_i, so that R_i is satisfied by I. In expanding I with α^i's, each antecedent of R_i in I is marked, then only literals in α^i filtered by I are added. In the case that R_i is $(L \rightarrow \varepsilon)$, L is simply marked and nothing is added with this production.

The ordinal powers of $\mathbf{T}_{\tau P}$ are defined as follows.

$$\begin{array}{lcl} \mathbf{T}_{\tau P} \uparrow 0 & = & \{\emptyset\}, \\ \mathbf{T}_{\tau P} \uparrow n+1 & = & \mathbf{T}_{\tau P}(\mathbf{T}_{\tau P} \uparrow n), \\ \mathbf{T}_{\tau P} \uparrow \omega & = & \bigcup_{\alpha < \omega} \bigcap_{\alpha \leq n < \omega} \mathbf{T}_{\tau P} \uparrow n, \end{array}$$

where n is a successor ordinal and ω is a limit ordinal. The above definition means that at the limit ordinal ω the closure retains transactions which are persistent in the preceding computation. That is, for any transaction I in $\mathbf{T}_{\tau P} \uparrow \omega$, there is an ordinal α smaller than ω such that, for every n $(\alpha \leq n < \omega)$, I is included in $\mathbf{T}_{\tau P} \uparrow n$. This closure definition is also used in (Inoue and Sakama, 1996) for computing minimal models of disjunctive logic programs. By the result in (Inoue and Sakama, 1996), $\mathbf{T}_{\tau P} \uparrow \omega$ becomes a fixpoint.

Example 3.2 Consider the transaction program τP in Example 3.1. Let $\tau P^{+p} = \tau P \cup \{ \rightarrow in(p) \}$. Then, the fixpoint closure of τP^{+p} becomes

$$\begin{array}{lcl} \mathbf{T}_{\tau P^{+p}} \uparrow 1 & = & \{\, \{in(p)\} \,\}, \\ \mathbf{T}_{\tau P^{+p}} \uparrow 2 & = & \{\, \{in(p)*, in(q), out(a)\}, \\ & & \quad \{in(p)*, in(b), out(r)\} \,\}, \\ \mathbf{T}_{\tau P^{+p}} \uparrow 3 & = & \{\, \{in(p)*, in(q)*, out(c), out(a)*\}, \\ & & \quad \{in(p)*, in(b), out(r)*, out(d)\} \,\}, \\ \mathbf{T}_{\tau P^{+p}} \uparrow 4 & = & \mathbf{T}_{\tau P^{+p}} \uparrow 3. \end{array}$$

Thus, $\mathbf{T}_{\tau P^{+p}} \uparrow \omega = \mathbf{T}_{\tau P^{+p}} \uparrow 3$.

Note that the above fixpoint computation constructs minimal models in a *bottom-up* manner when productions are viewed as disjunctive clauses. *Model generation procedures* such as SATCHMO, MGTP, and Hyper-Tableaux used in (Bry, 1990; Inoue and Sakama, 1996; Aravindan and Baumgartner, 1997) are similar to this computation mechanism. They perform hyperresolution to expand Herbrand interpretations and case-splitting on non-unit hyperresolvents. When a program is finite, function-free and range-restricted,[3] closure computation by these procedures

[3] A program P is *range-restricted* if for every rule in P, any variable appearing in the head has an occurrence in a positive atom in the body.

stops in a finite step. For a ground transaction program, these conditions are always satisfied.

We are ready to compute extended abduction from the fixpoint of a transaction program. T-literals $in(A)/out(A)$ with $A \in \mathcal{A}$ are called *abducible literals*. For a transaction S, the set of *unmarked abducible literals* in S is denoted as S°. For a set of transactions $\mathbf{I} \subseteq 2^{\mathcal{B}}$, let

$$min^\circ(\mathbf{I}) = \{ S^\circ \mid S \in \mathbf{I} \text{ and there is no } S' \in \mathbf{I} \text{ such that } S'^\circ \subset S^\circ \}.$$

Let us denote a transaction program with an observation G to be (un)explained as:

$$\tau P^{+G} = \tau P \cup \{ \rightarrow in(G),\ \rightarrow out(\bot) \},$$
$$\tau P^{-G} = \tau P \cup \{ \rightarrow out(G),\ \rightarrow out(\bot) \}.$$

Here, $out(\bot)$ means that all integrity constraints should be satisfied. When there is no integrity constraint in P, $(\rightarrow out(\bot))$ is not added to τP^{+G} and τP^{-G}. Also, we write

$$IN(S) = \{A \mid in(A) \in S\},$$
$$OUT(S) = \{A \mid out(A) \in S\}.$$

Definition 3.3 For an acyclic ALP $\langle P, \mathcal{A} \rangle$ and a ground observation G, *abduction steps in* P are inductively defined as follows.

1. G is *explainable by* $(\emptyset, \emptyset)$ *at 0-step* if $G \in P$.
   G is *explainable by* $(\{G\}, \emptyset)$ *at 0-step* if $G \in \mathcal{A}$.

2. G is *unexplainable by* $(\emptyset, \emptyset)$ *at 0-step* if P has no ground rule from P with G in the head.
   G is *unexplainable by* $(\emptyset, \{G\})$ *at 0-step* if $G \in P$ and $G \in \mathcal{A}$.

3. G is *explainable by* (E, F) *at* $(s+1)$*-step* if there is a ground rule:

   $$G \leftarrow A_1, \ldots, A_m, \mathit{not}\, A_{m+1}, \ldots, \mathit{not}\, A_n$$

   from P such that (i) each A_i $(i = 1, \ldots, m)$ is explainable by (E_i, F_i) at l_i-step $(l_i \leq s)$ and each A_j $(j = m, \ldots, n)$ is unexplainable by (E_j, F_j) at l_j-step $(l_j \leq s)$, (ii) $E = \bigcup_{i=1}^{m} E_i$, $F = \bigcup_{j=1}^{n} F_j$, $E \cap F = \emptyset$, and (iii) $max(l_1, \ldots, l_m, l_{m+1}, \ldots, l_n) = s$.

4. G is *unexplainable by* (E, F) *at* $(s+1)$*-step* if for every ground rule r_k of the form:

   $$G \leftarrow A_1, \ldots, A_{m_k}, \mathit{not}\, A_{m_k+1}, \ldots, \mathit{not}\, A_{n_k}$$

   from P, (i) there is an atom A_{i_k} $(1 \leq i_k \leq n_k)$ such that A_{i_k} is unexplainable (when $1 \leq i_k \leq m_k$) or explainable (when $m_k + 1 \leq i_k \leq n_k$) by (E_k, F_k) at l_k-step $(l_k \leq s)$, (ii) $E = \bigcup_{r_k} E_k$, $F = \bigcup_{r_k} F_k$, $E \cap F = \emptyset$, and (iii) $max_{r_k} l_k = s$.

In particular, we say that G is *(un)provable in* P at s-step for some $s < \omega$ if G is (un)explainable by $(\emptyset, \emptyset)$ in P at s-step. Note that this definition of provability (resp. unprovability) of G in P characterizes a positive (resp. negative) conclusion about G in P under the *well-founded semantics* (van Gelder *et al.*, 1991). Namely, G is true/false under the well-founded model of P iff G is provable/unprovable in P. Since the well-founded model and the stable model coincide in acyclic NLPs (van Gelder *et al.*, 1991), it holds that: for an acyclic NLP P, $P \models G$ (under the stable model semantics) iff G is provable in P at s-step.

Lemma 3.1 *Let* $\langle P, \mathcal{A} \rangle$ *be an acyclic ALP,* G *a ground observation, and* $E, F \in \mathcal{A}$. *Suppose that* $(P \cup E) \setminus F$ *is consistent.*

(i) *If* G *is (un)explainable by* (E, F) *in* P *at* s*-step for some* $s < \omega$*, then* (E, F) *is an (anti-)explanation of* G *wrt* $\langle P, \mathcal{A} \rangle$.

(ii) *If* (E, F) *is a minimal (anti-)explanation of* G *wrt* $\langle P, \mathcal{A} \rangle$*, then* G *is (un)explainable by* (E, F) *in* P *at* s*-step for some* $s < \omega$.

Proof: The proof of (i) is obvious by induction on the abduction step in P.

(ii) Let (E, F) be a minimal explanation of G wrt $\langle P, \mathcal{A} \rangle$. Then, $(P \cup E) \setminus F \models G$. By the discussion above, G is provable in $(P \cup E) \setminus F$ at s-step for some $s < \omega$. In other words, G is explainable by $(\emptyset, \emptyset)$ in $(P \cup E) \setminus F$ at s-step. Then, G is explainable by $(\emptyset, F)$ in $P \cup E$ at s-step, because (E, F) is a minimal explanation of G and hence no $F' \subset F$ satisfies that G is explainable by $(\emptyset, F')$ in $P \cup E$. Then, G is explainable by (E, F) in P at s-step, because (E, F) is a minimal explanation of G and hence no $E' \subset E$ satisfies that G is explainable by (E', F) in P.

The proof for a minimal anti-explanation can be shown in a similar way. □

Theorem 3.2 *Let* $\langle P, \mathcal{A} \rangle$ *be a ground acyclic ALP, and* G *a ground observation. Then,*

(i) (E, F) *is a minimal explanation of* G *wrt* $\langle P, \mathcal{A} \rangle$ *iff there is a transaction* $S \in min^\circ(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$ *such that* $E = IN(S)$ *and* $F = OUT(S)$.

(ii) (E, F) *is a minimal anti-explanation of* G *wrt* $\langle P, \mathcal{A} \rangle$ *iff there is a transaction* $S \in min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$ *such that* $E = IN(S)$ *and* $F = OUT(S)$.

Proof: We prove that G is (un)explainable by (E, F) at s-step for some $s < \omega$ iff there is a transaction S in $min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$ (or $min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$) such that $E = IN(S)$ and $F = OUT(S)$. This is proved by induction on the abduction step s. Then, the results (i) and (ii) follow by Lemma 3.1. Suppose first that $s = 0$.

(i) If G is explainable by (E, F) at 0-step, (E, F) is either $(\emptyset, \emptyset)$ or $(\{G\}, \emptyset)$. In the former case, $G \in P$ holds, and then $(in(G) \rightarrow \varepsilon)$ is in τP^{+G}. Hence, $min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega) = \{\emptyset\}$, $E = IN(\emptyset)$ and $F = OUT(\emptyset)$. In the latter case, $G \in \mathcal{A}$ holds, and then $\{in(G)\} \in min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$, $E = IN(\{in(G)\})$ and $F = OUT(\{in(G)\})$.

(ii) If G is unexplainable by (E, F) at 0-step, (E, F) is either $(\emptyset, \emptyset)$ of $(\emptyset, \{G\})$. In the former case, P has no rule with G in the head. Then, $(out(G) \rightarrow \varepsilon)$ is in τP^{-G}, hence $min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega) = \{\emptyset\}$, $E = IN(\emptyset)$ and $F = OUT(\emptyset)$. In the latter case, $G \in P \cap \mathcal{A}$ holds. Then, $\{out(G)\} \in min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$, $E = IN(\{out(G)\})$ and $F = OUT(\{out(G)\})$.

Next suppose that the results hold for every step $l \leq s$.

(i) Suppose that G is explainable by (E, F) at $(s+1)$-step, where (E, F) is a minimal explanation of G. Since P is acyclic, there is a ground rule

$$G \leftarrow A_1, \ldots, A_m, \mathit{not}\, A_{m+1}, \ldots, \mathit{not}\, A_n$$

in P such that (i) each A_i $(i = 1, \ldots, m)$ is explainable by (E_i, F_i) at l_i-step $(l_i \leq s)$ and each A_j $(j = m, \ldots, n)$ is unexplainable by (E_j, F_j) at l_j-step $(l_j \leq s)$, (ii) $E = \bigcup_{i=1}^{m} E_i$, $F = \bigcup_{j=1}^{n} F_j$, and $E \cap F = \emptyset$. By the induction hypothesis, each (E_i, F_i) is a minimal explanation of A_i iff $S_i \in min^{\circ}(\mathbf{T}_{\tau P^{+A_i}} \uparrow \omega)$ such that $E_i = IN(S_i)$ and $F_i = OUT(S_i)$. Likewise, each (E_j, F_j) is a minimal anti-explanation of A_j iff $S_j \in min^{\circ}(\mathbf{T}_{\tau P^{-A_j}} \uparrow \omega)$ such that $E_j = IN(S_j)$ and $F_j = OUT(S_j)$.

On the other hand, there is a production

$$in(G) \rightarrow in(B_1) \mid \ldots \mid in(B_l)$$

in τP such that for some $1 \leq k \leq l$, $in(B_k)$ is equal to

$$in(A_1), \ldots, in(A_m), out(A_{m+1}), \ldots, out(A_n).$$

Let $S = \{in(G)*\} \cup \bigcup_{i=1}^{n} S_i$. Here, $E \cap F = \emptyset$ implies that S is coherent. Then, $S \in \mathbf{T}_{\tau P^{+G}} \uparrow \omega$. Also, the minimality of each (E_i, F_i) implies the minimality of S°. Therefore, $S^{\circ} \in min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$ and $E = IN(S^{\circ})$ and $F = OUT(S^{\circ})$.

(ii) Suppose that G is unexplainable by (E, F) at $(s+1)$-step, where (E, F) is a minimal anti-explanation of G. Then, for every ground rule of the form

$$r_k : (G \leftarrow A_1, \ldots, A_{m_k}, \mathit{not}\, A_{m_k+1}, \ldots, \mathit{not}\, A_{n_k})$$

in P, there is an atom A_{i_k} $(1 \leq i_k \leq n_k)$ such that A_{i_k} is unexplainable (when $1 \leq i_k \leq m_k$) or explainable (when $m_k + 1 \leq i_k \leq n_k$) by (E_k, F_k) at l_k-step $(l_k \leq s)$, (ii) $E = \bigcup_{r_k} E_k$, $F = \bigcup_{r_k} F_k$, and $E \cap F = \emptyset$. By the induction hypothesis, each (E_k, F_k) is a minimal anti-explanation of A_{i_k} $(1 \leq i_k \leq m_k)$ iff $S_k \in min^{\circ}(\mathbf{T}_{\tau P^{-A_{i_k}}} \uparrow \omega)$ such that $E_k = IN(S_k)$ and $F_k = OUT(S_k)$. Likewise, each (E_k, F_k) is a minimal explanation of A_{i_k} $(m_k + 1 \leq i_k \leq n_k)$ iff $S_k \in min^{\circ}(\mathbf{T}_{\tau P^{+A_{i_k}}} \uparrow \omega)$ such that $E_k = IN(S_k)$ and $F_k = OUT(S_k)$.

On the other hand, there are productions

$$(out(G) \rightarrow out(B_1)),\ \ldots,\ (out(G) \rightarrow out(B_u))$$

in τP, where each $out(B_i)$ is a disjunction of literals. Then, these productions can be rewritten to one production with $out(G)$ in the antecedent:

$$out(G) \rightarrow out(B'_1) \mid \ldots \mid out(B'_v),$$

where each $out(B'_i)$ is now a conjunction of literals formed by collecting one disjunct from every disjunction $out(B_k)$ for $k = 1, \ldots, u$. Then, there is an $out(B'_h)$ which contains either $S_k \in min^{\circ}(\mathbf{T}_{\tau P^{-A_{i_k}}} \uparrow \omega)$ $(1 \leq i \leq m_k)$ or $S_k \in min^{\circ}(\mathbf{T}_{\tau P^{+A_{i_k}}} \uparrow \omega)$ $(m_k+1 \leq j \leq n_k)$ for every r_k. Let $S = \{out(G)*\} \cup \{out(B'_h)\}$. Here, $E \cap F = (\bigcup_{r_k} E_k) \cap (\bigcup_{r_k} F_k) = \emptyset$ implies that S is coherent. Then, by the fixpoint construction, $S \in \mathbf{T}_{\tau P^{-G}} \uparrow \omega$. Also, the minimality of each (E_k, F_k) implies the minimality of S°. Hence, $S^{\circ} \in min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$. By $E = \bigcup_{r_k} IN(S_k)$ and $F = \bigcup_{r_k} OUT(S_k)$, $E = IN(S^{\circ})$ and $F = OUT(S^{\circ})$.

To show the converse direction, put $E = IN(S)$ and $F = OUT(S)$ for $S \in min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$ (resp. $S \in min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$). Then, we can show that G is explainable (resp. unexplainable) by (E, F) at s-step by following the above proofs backward. □

Corollary 3.3 *A ground observation* G *has no explanation (resp. anti-explanation) in a ground acyclic ALP* $\langle P, \mathcal{A} \rangle$ *if* $\mathbf{T}_{\tau P^{+G}} \uparrow \omega = \emptyset$ *(resp.* $\mathbf{T}_{\tau P^{-G}} \uparrow \omega = \emptyset$*).*

Example 3.3 In Example 3.2, $min^{\circ}(\mathbf{T}_{\tau P^{+p}} \uparrow \omega) = \{\, \{out(c)\}, \{in(b), out(d)\} \,\}$. Then, $G = p$ has two minimal explanations $(E, F) = (\emptyset, \{c\}), (\{b\}, \{d\})$.

When a program contains cycles, Theorem 3.2 does not hold in general.

Example 3.4 Let P be the program

$$p \leftarrow q,$$
$$q \leftarrow p,$$

and $\mathcal{A} = \emptyset$. Then, p has no explanation. However,

$$in(p) \rightarrow in(q),$$
$$in(q) \rightarrow in(p)$$

are in τP, then $\mathbf{T}_{\tau P^{+p}} \uparrow \omega = \{\{in(p)*, in(q)\}\}$ and $min^{\circ}(\mathbf{T}_{\tau P^{+p}} \uparrow \omega) = \{\emptyset\}$, which does not imply the absence of explanation.

However, the acyclicity is not a necessary condition for Theorem 3.2.

Example 3.5 Let P be the program

$$g \leftarrow p,$$
$$p \leftarrow not\, q,$$
$$q \leftarrow q,$$

and $\mathcal{A} = \emptyset$. Then, g has the explanation $(E, F) = (\emptyset, \emptyset)$. In this case, τP includes

$$in(g) \rightarrow in(p),$$
$$in(p) \rightarrow out(q),$$
$$out(q) \rightarrow out(q).$$

Then, $\mathbf{T}_{\tau P^{+g}} \uparrow \omega = \{\{in(g)*, in(p)*, out(q)\}\}$, and $min^{\circ}(\mathbf{T}_{\tau P^{+g}} \uparrow \omega) = \{\emptyset\}$, which corresponds to the explanation.

3.3 NON-GROUND CASE

We extend results in previous subsections to non-ground programs. In this section, we consider a non-ground ALP $\langle P, \mathcal{A} \rangle$ in which P is a *covered* NLP and observations are ground. Here, $\langle P, \mathcal{A} \rangle$ is called a *covered* ALP if P is *covered*, that is, for every rule in P, all variables in the body appear in the head (Subrahmanian, 1987).[4] With this restriction, transactions in any bottom-up computation are guaranteed to be always ground. We also assume that for each predicate p, every atom with p is either abducible or non-abducible, and call such a predicate an *abducible predicate* in the former case. While we might consider a more general case using theories of predicate completion in logic programming, such an extension is too technical and is beyond the scope of this paper.

In the presence of variables in a program, we use *Clark's completion* (Clark, 1978) for producing transaction programs. Recall that Clark's completion is defined as follows. Any rule

$$p(t_1, \ldots, t_n) \leftarrow B$$

in P, where $t_1, \ldots, t_n$ are terms, is written in the form

$$p(x_1, \ldots, x_n) \leftarrow (\exists \mathbf{y})\,(x_1 = t_1), \ldots, (x_n = t_n),\ B,$$

where $x_1, \ldots, x_n$ are new variables and $\mathbf{y}$ are the variables in the original rule. Note here that when P is covered and each t_i is ground, $(\exists \mathbf{y})$ need not be considered. If there are k rules with the predicate p in the head, they are written in the form

$$\begin{array}{l} p(x_1, \ldots, x_n) \leftarrow E_1, B_1, \\ \quad \vdots \\ p(x_1, \ldots, x_n) \leftarrow E_k, B_k, \end{array} \tag{7}$$

where E_i's are equalities and B_i's are original bodies. Then, the definition for p is

$$p(x_1, \ldots, x_n) \leftrightarrow E_1, B_1 \mid \ldots \mid E_k, B_k.$$

Now, given an ALP $\langle P, \mathcal{A} \rangle$, its transaction program is defined like Definition 3.1 based on Clark's completion.

Definition 3.4 Let $\langle P, \mathcal{A} \rangle$ be a covered ALP. The *transaction program τP of P (relative to $\mathcal{A}$)* is defined as follows.

1. Let p be a non-abducible predicate and $k > 0$ for rules (7). Then, for $H = p(x_1, \ldots, x_n)$, rules (7) are transformed to productions:

$$R_{in}:\ in(H) \rightarrow E_1, in(B_1) \mid \ldots \mid E_k, in(B_k), \tag{8}$$

$$\begin{array}{ll} R_{out}: & out(H),\ E_1 \rightarrow out(B_1), \\ & \quad \vdots \\ & out(H),\ E_k \rightarrow out(B_k), \end{array} \tag{9}$$

$$out(H),\ \neg E_1, \ldots, \neg E_k \rightarrow \varepsilon, \tag{10}$$

   where for $i = 1, \ldots, k$, $in(B_i)$ and $out(B_i)$ are defined in the same way as Definition 3.1. In particular, if B_i is empty, then $in(B_i) = \varepsilon$ and $out(B_i) = false$. Also, $\neg E = (x_1 \neq t_1) \vee \ldots \vee (x_n \neq t_n)$ for $E = (x_1 = t_1), \ldots, (x_n = t_n)$.

   When $H = \bot$ (integrity constrains), there are no E_i's and only (9) in R_{out} is included in τP.

[4] A covered program is also called a *constrained* program in the literature, e.g., (Lavrač and Džeroski, 1994).

2. Let p be an abducible predicate and $k > 0$ for rules (7). In this case, every rule in (7) is in the form $p(t_1, \ldots, t_n) \leftarrow$ by the assumption.[5] Then, for $H = p(x_1, \ldots, x_n)$, rules (7) are transformed to

$$\begin{aligned} R_{in}: \quad & in(H),\ E_1 \rightarrow \varepsilon, \\ & \vdots \\ & in(H),\ E_k \rightarrow \varepsilon, \end{aligned} \tag{11}$$

$$R_{out}: \quad out(H),\ \neg E_1, \ldots, \neg E_k \rightarrow \varepsilon. \tag{12}$$

3. Let p be any predicate whose definition is empty, i.e., $k = 0$ for rules (7). For every such predicate p, introduce the production:

$$out(p(x_1, \ldots, x_n)) \rightarrow \varepsilon \tag{13}$$

to τP. Moreover, when p is a non-abducible predicate, τP also includes:

$$in(p(x_1, \ldots, x_n)) \rightarrow false. \tag{14}$$

An essential difference from the ground case is that transaction programs now contain equalities E_i's and inequalities $\neg E_i$'s. In a covered program with a ground observation, we can regard each equality $x = t$ or inequality $x \neq t$ just as a literal to be evaluated for comparison of terms: $x = t$ is *true* ($x \neq t$ is *false*) if the terms currently instantiating x and t are literally identical or they are unifiable; otherwise $x = t$ is *false* ($x \neq t$ is *true*). (In)equalities in antecedents of productions are thus evaluated for this checking after T-literals in antecedents are instantiated. Then, if x and t are unifiable in an equality $x = t$ in the antecedent of some production (9), every occurrence of t in T-literals in the consequent of the production is substituted with the term currently instantiating x. On the other hand, if x and t are unifiable in $x = t$ in the consequent of (8), every occurrence of t in the same disjunct of the consequent is substituted with the term currently instantiating x. Note that when an NLP P is ground, Definition 3.4 reduces to Definition 3.1.

The mapping $T_{\tau P}$ in Definition 3.2 is also slightly extended as follows. Variables in each production are instantiated with a substitution σ such that $in(H\sigma)$ or $out(H\sigma)$ in the antecedent is contained in a transaction I. Marking is performed upon an instantiated literal in the antecedent of a production. Here, in obtaining a substitution σ in the $T_{\tau P}$ operator, it is sufficient to consider *matching* instead of full unification if a given NLP is covered and an observation is ground. Starting from a ground observation $in(G)$ or $out(G)$, the fixpoint operator is repeatedly applied as long as some transaction is changed. In this case, every transaction I constructed in fixpoint computation contains only ground literals. Model generation procedures introduced in Section 3.2 can be used in this case too, since τP is range-restricted whenever P is constrained.

With this setting, fixpoint closures are defined in the same manner as in the preceding section, and $\mathbf{T}_{\tau P} \uparrow \omega$ reaches the fixpoint. Then, results of Theorem 3.2 are directly extended to computing (anti-)explanations from non-ground acyclic programs.

Theorem 3.4 *Let $\langle P, \mathcal{A} \rangle$ be a covered acyclic ALP, and G a ground observation. Then,*

(i) *(E, F) is a minimal explanation of G wrt $\langle P, \mathcal{A} \rangle$ iff there is a transaction $S \in min^{\circ}(\mathbf{T}_{\tau P^{+G}} \uparrow \omega)$ such that $E = IN(S)$ and $F = OUT(S)$.*

(ii) *(E, F) is a minimal anti-explanation of G wrt $\langle P, \mathcal{A} \rangle$ iff there is a transaction $S \in min^{\circ}(\mathbf{T}_{\tau P^{-G}} \uparrow \omega)$ such that $E = IN(S)$ and $F = OUT(S)$.*

Proof: The proof of Theorem 3.2 can be lifted to the non-ground case. □

Example 3.6 Let $\langle P, \mathcal{A} \rangle$ be an ALP, where

$$\begin{aligned} P: \quad & g(x) \leftarrow p(x),\ not\ q(x), \\ & q(a) \leftarrow, \\ \mathcal{A}: \quad & p(x),\ q(x). \end{aligned}$$

Then, τP includes

$$\begin{aligned} & in(g(y)) \rightarrow (y = x),\ in(p(x)),\ out(q(x)), \\ & out(g(y)),\ (y = x) \rightarrow out(p(x)) \mid in(q(x)), \\ & out(p(x)) \rightarrow \varepsilon, \\ & in(q(x)),\ (x = a) \rightarrow \varepsilon, \\ & out(q(x)),\ (x \neq a) \rightarrow \varepsilon. \end{aligned}$$

Let $G1 = g(a)$ be an observation. By $min^{\circ}(\mathbf{T}_{\tau P^{+G1}} \uparrow \omega) = \{\{in(p(a)), out(q(a))\}\}$, the minimal explanation of $G1$ is $(\{p(a)\}, \{q(a)\})$.

Next, let $G2 = g(b)$ be another (negative) observation. Then, $\mathbf{T}_{\tau P^{-G2}} \uparrow \omega$ includes $\{out(g(b))*, out(p(b))*\}$ and $\{out(g(b))*, in(q(b))\}$, so $min^{\circ}(\mathbf{T}_{\tau P^{-G2}} \uparrow \omega) = \{\emptyset\}$. Hence, $(\emptyset, \emptyset)$ is the minimal anti-explanation of $G2$.

[5] Recall that any rule with an abducible in the head is always a fact (see Section 2).

4 DISCUSSION

4.1 ABDUCTIVE PROCEDURE

The idea of computing abduction through transaction programs is close to that of computing abduction through *completion* (Console *et al.*, 1991; Konolige, 1992). Our approach is different from them in the following points. First, our procedure is intended to compute positive, negative and mixed (anti-)explanations in extended abduction, while (Console *et al.*, 1991; Konolige, 1992) compute positive explanations only. In fact, productions R_{out} are newly introduced in our transformation. Second, we generate a transaction program which declaratively specifies an abductive procedure, while no such program is generated in those work. In (Console *et al.*, 1991), soundness and completeness are guaranteed for hierarchical NLPs. In our procedure, they are guaranteed for acyclic NLPs.

A fixpoint semantics of abductive logic programs is also introduced in (Inoue and Sakama, 1996), but it computes traditional abduction and does not compute the extended one. However, we have shown in this paper that a similar fixpoint operator can be used to compute extended abduction. In fact, a transaction program is a kind of disjunctive programs and fixpoint computation can be realized by model generation procedures used in (Bry, 1990; Inoue and Sakama, 1996; Aravindan and Baumgartner, 1997).

4.2 DATABASE UPDATE

It is known that abduction is used for *database update*. In deductive databases, an update request on an intensional fact is often translated into update on extensional facts. Such database update is called *view update*. More precisely, the view update problem is presented as follows. A *deductive database* (or *database* for short) is defined as a function-free consistent NLP. Let D be a database and $\mathcal{E}$ a set of ground facts in the language of D called *extensional facts*. Then, an *insertion* of a ground fact A into D (resp. *deletion* of A from D) is accomplished by an *updated database*

$$D' = (D \cup E) \setminus F \quad \text{with} \quad E, F \subseteq \mathcal{E}$$

satisfying the conditions:

1. $D' \models A$ (resp. $D \not\models A$) where D' is consistent,
2. there is no database D'' satisfying the condition 1 and $D \sim D'' \subset D \sim D'$, where $D_1 \sim D_2 = (D_1 \setminus D_2) \cup (D_2 \setminus D_1)$.

The first condition says that an updated database accomplishes an insertion/deletion, while the second condition presents that the update *minimally changes* extensional facts stored in D.

Relating to abduction, an update request corresponds to an observation and extensional facts correspond to abducibles. Then, view update can be characterized by extended abduction.

Lemma 4.1 (Inoue and Sakama, 1995) *Given a database D and a ground atom A, an updated database D' accomplishes an insertion/deletion of A into/from D iff there is a minimal explanation/anti-explanation (E, F) of A in D.*

Using this lemma, the next results are obtained by Theorem 3.2.

Theorem 4.2 *Let D be a covered acyclic database, and A a ground atom.*

(i) *An insertion of A into D is achieved by an update (E, F) iff there is a transaction $S \in min^{\circ}(\mathbf{T}_{\tau D^{+A}} \uparrow \omega)$ such that $E = IN(S)$ and $F = OUT(S)$.*

(ii) *A deletion of A from D is achieved by an update (E, F) iff there is a transaction $S \in min^{\circ}(\mathbf{T}_{\tau D^{-A}} \uparrow \omega)$ such that $E = IN(S)$ and $F = OUT(S)$.*

Thus, we can compute database update via fixpoint computation of a transaction program.

Kakas and Mancarella (1990) characterize database update through abduction, and use a top-down abductive procedure for computing view update. In (Bry, 1990), abduction is translated into a disjunctive program and database update is realized by bottom-up computation on a meta-program specifying an update procedure. These procedures are based on traditional abduction and do not produce transaction programs.

In database update, Rossi and Naqvi (1989) use Clark's completion to compute update in Horn databases. Given an update request, it is transformed to non-Horn formulas obtained by the only-if parts of database clauses. The resulting formulas represent update on the original database. Such formulas are computed using an SLD-like top-down procedure, while they do not generate transaction programs.

In (Grant *et al.*, 1993), view update is translated into a set of disjunctive facts. This translation is based on expansion of an SLD-tree, and update is achieved by inserting/deleting these disjunctive facts to/from

a database. In (Fernández *et al.*, 1996), update is achieved through construction of minimal models that satisfy an update request. In these work, databases are treated as ground programs possibly containing disjunctive facts, but transaction programs are not produced. Our transaction program specifies update by disjunction but does not introduce disjunctions to a database.

A transaction program is also viewed as a database update language which declaratively specifies dynamic changes in databases. Our program transformation provides an *automatic generation of update specification* from the original program. The generated transaction program is a kind of disjunctive program, and non-determinism in database update is naturally expressed using disjunctive productions. In this sense, transaction programs provide a new application of disjunctive logic programming. Note that several database update languages exist (Abiteboul, 1988), but few consider disjunctions in the language.

4.3 VIEW DELETION FROM DEFINITE DATABASE

Recently, Aravindan and Baumgartner (1997) proposed a transformation of ground definite programs into disjunctive programs to compute view deletion. For definite programs, R_{out} in our transaction programs is similar to their transformation, and can be used to realize view deletion in computing anti-explanations. Like (Aravindan and Baumgartner, 1997), the correctness of our fixpoint computation is guaranteed also for *non-acyclic* Horn programs.

Theorem 4.3 *Let $\langle P, \mathcal{A} \rangle$ be a ground Horn ALP, and G a ground observation. Then, F is a minimal negative anti-explanation of G wrt $\langle P, \mathcal{A} \rangle$ iff there is a transaction $S \in min^{\circ}(\mathbf{T}_{\tau P - G} \uparrow \omega)$ such that $F = OUT(S)$.*

Proof: By Lemma 4.1, a deletion of G is accomplished iff there is a minimal anti-explanation (E, F) of G wrt $\langle P, \mathcal{A} \rangle$. Since P is Horn, negation as failure does not appear and hence $E = \emptyset$, that is, F is a negative anti-explanation of G. Moreover, R_{out} in Definition 3.1 consists of productions containing *out*-literals only. Since an update request is deletion of G, only *out*-literals are concerned and R_{in} can be ignored in the transaction program τP.

Now, let S be any transaction in $\mathbf{T}_{\tau P - G} \uparrow \omega$, and $F = OUT(S^{\circ})$. Also, let $out(L)$ be any unmarked literal in S. There are two cases for L:

1. *L is an abducible and $(L \leftarrow) \in P$.*
   In this case, L actually has to be removed from P.

2. *L is not an abducible.*
   In this case, L has a non-empty definition in P, because otherwise, $(out(L) \rightarrow false)$ is in τP and S is not included in the fixpoint. In this case, a production of the form:

   $$out(L) \rightarrow out(A_1) \mid \ldots \mid out(A_m)$$

   is in τP, and for some A_i $(1 \leq i \leq m)$, either $out(A_i)$ or $out(A_i)*$ is included in S. When $out(A_i) \in S$, L's removal is accomplished by the deletion of A_i. When $out(A_i)* \in S$, L's removal has already been translated into either ε when A_i has no definition, or otherwise a request of deletion of some other literals.

In the second case above, we can consider again the above two cases for A_i. If A_i is an abducible (i.e., the first case above), L's removal is successfully accomplished. Otherwise, by recursively analyzing the second case above, the chain eventually terminates as $L, L_1(= A_i), \ldots, L_k$, where L_k is either an abducible or identical to L, since the number of rules in P is finite. If L_k is an abducible, we can utilize the definition of abduction steps in Definition 3.3, so that L is unexplainable at k-step and Theorem 3.2 can be applied as well. Else if $L_k = L$, the program P is not acyclic. In this case, L_k cannot be entailed from P, and thus L_k can be unexplained and so can be L. Therefore, $out(L)$ for a non-abducible L can be removed from S by taking S°, and hence $F = OUT(S^{\circ})$ is a negative anti-explanation of G. The minimality of anti-explanations is also guaranteed by taking the operation min° to the fixpoint.

The proof of the converse direction can also be shown based on a similar argument as above. □

As (Aravindan and Baumgartner, 1997) pointed out, a general *contraction* algorithm including view deletion for definite programs requires computation of (i) all minimal positive explanations of the sentence to be contracted wrt the background knowledge without abducible facts, and then (ii) a (minimal) *hitting set* (Reiter, 1987) for these explanations, which corresponds to a (minimal) negative anti-explanation. Both Aravindan and Baumgartner's method and ours directly compute such hitting sets without computing positive explanations explicitly. In contrast to (Aravindan and Baumgartner, 1997), our transaction programs can be applied to programs with negation as failure, and can also be used to compute positive (negative, and mixed) (anti-)explanations in extended abduction.

4.4 REVISION PROGRAM

Marek and Truszczynski (1994) define a language to specify revision in knowledge bases. The meaning of their *revision programs* is similar to the stable model semantics for logic programming. They define a *knowledge base* (KB) as a set of propositional atoms. A *(disjunctive) revision program* Π consists of *revision rules* of the form:

$$in(p_1) \mid \ldots \mid in(p_k) \mid out(r_1) \mid \ldots \mid out(r_l) \\ \leftarrow in(q_1), \ldots, in(q_m), out(s_1), \ldots, out(s_n), \quad (15)$$

where each of p_i, r_i, q_i, s_i is a propositional atom. Each revision rule in Π can be regarded as a clause in propositional logic. Then, for each minimal model M of Π, a pair (I, O) where $I = \{a \mid in(a) \in M\}$ and $O = \{a \mid out(a) \in M\}$ is called a *necessary change* for Π. Let D_I (called an *initial KB*) and D_R be two KBs. The revision program $\Pi_{D_R}|D_I$ is obtained by:

1. deleting from Π every rule of the form (15) such that $q_i \notin D_R$ for some $1 \leq i \leq m$ or $s_j \in D_R$ for some $1 \leq j \leq n$, and
2. deleting each $in(a)$ such that $a \in D_I$ and each $out(a)$ such that $a \notin D_I$ from the body of every remaining revision rule.

Let (I, O) be a necessary change for $\Pi_{D_R}|D_I$. Then, D_R is called a Π-*justified revision* of D_I if $I \cap O = \emptyset$ and $D_R = (D_I \cup I) \setminus O$. It has been shown that a Π-justified revision always performs a minimal change that is justified by the use of revision rules.

Although the direction of arrows of revision rules ($\leftarrow$) and that of productions ($\rightarrow$) are opposite, they look very similar in their syntax. In fact, a transaction program is a *context-free* revision program, i.e., a disjunctive revision program in which every revision rule has only one literal in the body. While a production of the form (2) may have a disjunctive normal form formula in the consequent, it can be equivalently written in productions with a disjunction of literals in their consequents (Inoue and Sakama, 1996), and hence corresponds to multiple revision rules.

There are several differences between the framework of (Marek and Truszczyński, 1994) and ours. First, in revision programming, a KB contains no rules except facts, and there are no notion of abducibles. Instead, a necessary change represents a whole change of atoms included in the KB, and every atom in the language can be both inserted and deleted. Second, we used the notion of marking to represent literals whose transactions have been translated into another atoms' requests, but this information should also be ignored. This is because in revision programming every atom is subject to change and view update is not considered.

Taking these differences into account, we can utilize transaction programs in revision programming. As in Section 4.2, a typical application of revision programming is database update. Recall that a database is specified by an NLP P, which is divided into the intensional part (IDB) P_r containing the rules with non-empty bodies in P, and the extensional part (EDB) containing the facts in P. An initial KB D_I is then set to a stable model of P, which includes all facts in EDB. A revision program Π is constructed from P_r and an update request by transforming them to the transaction program ${\tau P_r}^{+G}$ or ${\tau P_r}^{-G}$. Then, a revised KB D_R is computed through the transaction program. Notice that D_R still satisfies IDB of P and minimally changes EDB.

Example 4.1 Let P be the NLP:

$$p(x) \leftarrow s(x), \mathit{not}\, q(x), \\ q(a) \leftarrow, \\ s(a) \leftarrow, \\ s(b) \leftarrow .$$

Here, the first rule is transformed to the productions:

$$R: \quad in(p(y)) \rightarrow (y = x),\ in(s(x)),\ out(q(x)), \\ out(p(y)),\ (y = x) \rightarrow out(s(x)) \mid in(q(x)).$$

Set the initial KB to the stable model of P, i.e.,

$$D_I = \{s(a), s(b), q(a), p(b)\}.$$

Let us consider insertion of $G = p(a)$, and put

$$\Pi = R \cup \{\ \rightarrow in(p(a))\ \}.$$

Viewing Π as a revision program, let

$$D_R = \{s(a), s(b), p(a), p(b)\}.$$

Then, we obtain

$$\Pi_{D_R}|D_I: \quad in(s(a)),\ out(q(a)) \leftarrow , \\ in(s(b)),\ out(q(b)) \leftarrow , \\ in(p(a)) \leftarrow ,$$

and the necessary change for $\Pi_{D_R}|D_I$ is $(I, O) = (\{p(a), s(a), s(b)\}, \{q(a), q(b)\})$. Hence,

$$D_R = (D_I \cup I) \setminus O$$

holds, and D_R is a Π-justified revision of D_I.

On the other hand, viewing Π as a transaction program, $S = \{in(p(a))*, in(s(a)), out(q(a))\}$ is included in $\mathbf{T}_\Pi \uparrow \omega$. By ignoring marking in S, we obtain $(I', O') = (\{p(a), s(a)\}, \{q(a)\})$. Hence,

$$D' = (D_I \cup I') \setminus O' = \{s(a), s(b), p(a), p(b)\},$$

which coincides with D_R in this case.

Note in the above example that if we do not ignore marking, we get $(IN(S), OUT(S)) = (\{s(a)\}, \{q(a)\})$, which is a smaller explanation of G. Furthermore, our framework can treat extensional facts as abducibles, thereby extends revision programming with *view*.

Hence, our transformation of P into τP provides a method of generating a revision program from the original program P. In deductive databases, such a program P is constructed from IDB. In other application domains, a program P can be any background knowledge base including causal relations, constraints, general laws and commonsense. As far as the authors know, there have been no other methods to generate revision programs automatically, and few discussion has been done on how to get revision programs.

5 CONCLUSION

This paper presented a method of computing extended abduction. An abductive logic program was transformed to a transaction program which declaratively specifies non-determinism in abduction. Then abductive explanations can be computed by the fixpoint of a transaction program, which is sound and complete for acyclic ALPs. In the context of databases, a transaction program provides a declarative specification of database update, and enables us to compute view update in a constructive manner. A transaction program is a kind of disjunctive programs and fixpoint computation is realized by model generation procedures.

This paper assumed acyclic and covered NLPs for programs containing variables. Future work includes relaxing these conditions and exploiting further applications of extended abduction.

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